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Cauchy problem for non-strictly hyperbolic systems II. Leray-Volevich's systems and well-posendness

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Introduction

We consider the Cauchy prolem for non strictly hyperbolic systems with diagonal principal part of constant multiplicity. We shall derive a necessary condition in order that the Cauchy problem for such systems is well posed in C^∞ class.

We consider the following Cauchy problem in $G(x)$ a neigborhood of $\hat{x}=(\hat{x}_0, \hat{x}_1,\dots, \hat{x}_n)\in \boldsymbol{R}^{n+1}$,

$$(1)\qquad \begin{cases} a(x, D)u^s(x)+\sum_{t=1}^{N} b_t^s(x, D)u^t(x)=f^s(x),\ x\in G(\hat{x})\cap\{x_0>\hat{x}_0\}, \\ D_0^h u^s|_{x_0=\hat{x}_0}=g_h^s(x'),\ x'\in G(\hat{x})\cap\{x_0=\hat{x}_0\},\quad h\leq m-1,\ s=1,\dots, N. \end{cases}$$

where $a(x, D)$ and $b_t^s(x, D)$ are differential operators of which coefficients are infinitely differential functions defined in a domain $G\subset R^{n+1}$. We assume here that we can factorize in $G\subset R^{n+1}$ the principal part of $a(x, D)$, $\hat{a}(x, \xi)$ as follows

$$(2)\qquad \hat{a}(x, \xi)=\prod_{l=1}^{r}(\xi_0-\lambda^{(l)}(x, \xi'))^{\nu^{(l)}},$$

where $\nu^{(l)}$ are constant integers in $G\times\boldsymbol{R}^n\backslash 0$, $\lambda^{(l)}$ are C^∞-real valued functions and $\lambda^{(l)}\neq\lambda^{(j)}$ on $G\times\boldsymbol{R}^n\backslash 0$ for $l\neq j$. Moreover we assume that there exist integers $n_1,\dots, n_N$ such that

$$(3)\qquad \text{order}\quad b_t^s\leq m-1+n_t-n_s$$

where $m=\text{order } a=\sum_{l=1}^{r}\nu^{(l)}$.

We call here a system with above properties (2) and (3) a hyperbolic Leray-Volevich's system with diagonal principal part of constant multiplicity.