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On the mollifier approximation for solutions of stochastic differential equations

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§ 0. Introduction

Consider the following stochastic differential equation (SDE) on $\boldsymbol{R}^d$

$$(0.1)\quad \begin{cases} dx_t^i = \sum_{\beta=1}^{r} \sigma_\beta^i(X(t))\circ dW^\beta(t) + b^i(X(t))dt \\ \quad = \sum_{\beta=1}^{r} \sigma_\beta^i(X(t))\cdot dW^\beta(t) + \left[\frac{1}{2}\sum_{j=1}^{d}\sum_{\beta=1}^{r}\left(\frac{\partial\sigma_\beta^i}{\partial x^j}\sigma_\beta^j\right)(X(t)) + b^i(X(t))\right]dt, \\ X(0) = x\in\boldsymbol{R}^d \qquad i=1, 2,\dots, d \end{cases}$$

with sufficiently smooth functions $\sigma_\beta^i(x)$ and $b^i(x)$ on $\boldsymbol{R}^d$. Here, $\circ dW^\beta(t)$ and $\cdot dW^\beta(t)$ denote the stochastic differentials of the *Stratonovich type* and of the *Itô type* respectively, and $W(t)=W(t, w)=(W^\beta(t))$, where $W(t, w)=w(t)$, $w\in W_0^r$, is the canonical realization of the r-dimensional Wiener process on the r-dimensional Wiener space $(W_0^r, \boldsymbol{P}^W)$: W_0^r is the space of all continuous functions w: $[0, \infty)\to\boldsymbol{R}^d$ such that $w(0)=0$ and $\boldsymbol{P}^W$ is the r-dimensional Wiener measure on W_0^r. Introducing vector fields $A_0, A_1,\dots, A_r$ on $\boldsymbol{R}^d$ by

$$A_\beta(x) = \sum_{i=1}^{d}\sigma_\beta^i(x)\frac{\partial}{\partial x^i}, \qquad \beta=1, 2,\dots, r$$

$$A_0(x) = \sum_{i=1}^{d} b^i(x)\frac{\partial}{\partial x^i},$$

the equation (0.1) is also denoted by

$$(0.1)'\quad \begin{cases} dX(t) = \sum_{\beta=1}^{r} A_\beta(X(t))\circ dW^\beta(t) + A_0(X(t))dt \\ X(0) = x. \end{cases}$$

If $\sigma_\beta^i(x)$ and $b^i(x)$ are $\boldsymbol{C}^\infty$ with bounded derivatives of all orders, the solution $X(t, x, w)$ exists globally and for $a.a.w(\boldsymbol{P}^W)$, $x\to X(t, x, w)$ is a diffeomorphism of $\boldsymbol{R}^d$ for each $t\geqq 0$ (cf. [1], [3]).