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Outradii of Teichmüller spaces of finitely generated Fuchsian groups of the second kind

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

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§ 1. Introduction.

A Fuchsian group Γ is said to be of the first kind (resp. second kind) if its region of discontinuity is not connected (resp. connected). The outradius $o(\Gamma)$ which is defined in §2 is strictly greater than 2 (Earle [7]) and not greater than 6 (Nehari [13]). This constant 6 cannot be replaced by any smaller one (Chu [6], Kalme [9]). In [14] the former author proved that $o(\Gamma)$ is strictly less than 6 for a finitely generated Fuchsian group Γ of the first kind. In this paper we prove the following.

Theorem. *If Γ is a finitely generated Fuchsian group of the second kind, then $o(\Gamma)$ is equal to* 6.

This theorem answers a question raised by Lipman Bers to the former author in U.S. -Japan Seminar on Kleinian Groups and Riemann Surfaces which was held at the East-West Center in Honolulu, Hawaii, during January 15-19, 1979. In §3 we state three lemmas without proofs. A proof of Theorem is given in §4. The rest of this note is devoted to prove lemmas stated in §3.

The authors would like to express their hearty thanks to Professor L. Bers for his kind indication of the problem to them.

§ 2. Definitions and notations.

Let $\hat{C}$ be the Riemann sphere. Let Δ be the open unit disc and Δ^* be the exterior of Δ in $\hat{C}$. Let $j(z)=1/\bar{z}$ be the reflection in $\partial\Delta$. For each μ in the open unit ball $L_\infty(\Delta)_1$ of $L_\infty(\Delta)$ we define two quasiconformal automorphisms w_μ and w^μ of $\hat{C}$. Let w_μ be the unique quasiconformal automorphism of $\hat{C}$ with fixed points $1, \sqrt{-1}$ and -1 which is μ-conformal in Δ and which satisfies $w_\mu \circ j = j \circ w_\mu$. In particular, w_μ keeps Δ invariant. Let w^μ be the unique quasiconformal automorphism of $\hat{C}$ with fixed points 1, $\sqrt{-1}$ and -1 which is μ-conformal in Δ and conformal in Δ^*.

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