J. Math. Kyoto Univ. (JMKYAZ)
26-2 (1986) 191-202

De Rham-Hodge-Kodaira's decomposition on an abstract Wiener space

By

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1. Introduction.

Recently a calculus on the Wiener space was developped by many authors and it is called Malliavin's calculus after his excellent papers [4], [5]. In this paper, we discuss differential forms on the Wiener space, or more generally, on an abstract Wiener space. Our discussion is based on Malliavin's calculus or more precisely on Sobolev spaces on an abstract Wiener space which are much due to P. A. Meyer [6] and arranged by H. Sugita [8]. We mainly follow Sugita [8] but we use a few different notations from those of Sugita.

Let (B, H, μ) be an abstract Wiener space; B is a (real) separable Banach space with the norm $\|\cdot\|_B$, H is a (real) separable Hilbert space that is densely and continuously imbedded in B with the inner product $\langle\cdot, \cdot\rangle_H$ and the norm $|\cdot|_H=\sqrt{\langle\cdot, \cdot\rangle_H}$ and μ is the Wiener measure, i.e., a Borel probability measure with the characteristic functional given by

$$\int_B e^{\sqrt{-1}(l, x)}\mu(dx)=\exp\left\{-\frac{1}{2}|l|_{H^*}^2\right\} \qquad l\in B^*\subseteqq H^*$$

where B^* is the dual space of B, H^* is the dual space of H with the norm $|\cdot|_{H^*}$ and $(\ ,\ )$ is the natural bilinear form on $B^*\times B$.

We define *differential forms* on an abstract Wiener space (see the section 2 for the definition), denoted by $\Lambda^n(B)$ where n is the degree of differential forms. As in the case of finite dimensional Riemannian manifold, we can define the *exterior derivative* d and its *dual operator* d^* and *de Rham-Hodge-Kodaira's Laplacian* Δ by

$$\Delta=dd^*+d^*d\,.$$

Then $\omega\in\Lambda^n(B)$ is called *hamonic* if

$$\Delta\omega=0$$

and we denote the set of all harmonic differential forms of degree n by $\mathfrak{h}_n$. We obtain the following decomposition:

$$\Lambda^n(B)=\mathrm{Im}(d)\oplus\mathrm{Im}(d^*)\oplus\mathfrak{h}_n \qquad \text{(direct sum)}$$

Communicated by Prof. Watanabe, November 2, 1984.