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On BMO property for potentials on Riemann surfaces

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Introduction. In this paper we shall deal with the BMO property for Green potentials on $\boldsymbol{R}^n$, on the unit disk D in the complex plane and on Riemann surfaces. For the functions on D, one can consider the following two BMO spaces naturally;

(1) BMO space with respect to the 2-dimensional Lebesgue measure on D.

(2) BMO space with respect to the hyperbolic measure on D.

The first space, more generally, BMO space on a domain in $\boldsymbol{R}^n$ with respect to the n-dimensional Lebesgue measure has been investigated in various points (for instance [2], [7], [10]). While it seems that the second one is not familiar and actually the studies of this hyperbolic BMO space can be found only in [6], [8], [10]. The hyperbolic BMO space was introduced by H. M. Reimann and T. Rychener [10], and they proved there it is contained in the first BMO space and asked whether or not these two BMO spaces coincide with each other. Recently, Y. Gotoh [6] gave the negative answer for this question by showing that for analytic functions on D, the hyperbolic BMO property coinsides with the usual BMO property of their boundary functions. Y. Kusunoki and M. Taniguchi [8] investigated these two BMO properties for potentials on D and also on Riemann surfaces. They showed that on Riemann surface the potential of positive measure with compact support belongs to hyperbolic BMO space. One of the main aims of this paper is to extend this result.

First in §1 we shall give a characterization of measure μ for which the Newtonian potential $\int_{\boldsymbol{R}^n} |x-y|^{-(n-2)}d\mu(y)$ should belongs to $BMO(\boldsymbol{R}^n)$.

In §2 we shall investigate the fundamental property of potentials on D which belong to BMO space. We shall show certain relation between Carleson measure and BMO property of potential and show that every potneital with finite energy, further every Dirichlet function belong to hyperbolic BMO space. Also we shall show that Riesz' decomposition preserves hyperbolic BMO property (see Theorem [6]) and on the other hand, it does not preserve the first BMO property.

The next §3 is concerned with BMO property for the potentials on Riemann surfaces. By applying the results obtained in §2, we shall show that on a compact bordered Riemann surface the potentials of finite measures and potentials of finite energy belongs to hyperbolic BMO space.

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