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Projective structures on Riemann surfaces and Kleinian groups

By

Hiroshige SHIGA

§1. Introduction and notations.

Let S be a compact Riemann surface of genus $p \geqq 2$, and let $\pi: U \to S$ be a holomorphic universal covering of S with the covering transformation group Γ, where U is the upper half plane $\{z \in \boldsymbol{C}: \operatorname{Im} z > 0\}$. Then, Γ is a finitely generated Fuchsian group of the first kind on U and consists of hyperbolic Möbius transformations. We denote by $B_2(L, \Gamma)$ the Banach space of all holomorphic quadratic differentials for Γ defined on the lower half plane L. Namely, $B_2(L, \Gamma)$ is the set of all holomorphic functions ϕ on L satisfying

$$\phi(\gamma(z))\gamma'(z)^2 = \phi(z), \qquad \text{for all } z \in L, \gamma \in \Gamma, \tag{1.1}$$

with the norm

$$\|\phi\|_L = \sup_{z \in L} (2 \operatorname{Im} z)^2 |\phi(z)| .$$

More generally, for a Kleinian group G and for a G-invariant union Δ of components of G we denote by $B_2(\Delta, G)$ the Banach space consisting of all holomorphic functions ψ on Δ satisfying

$$\psi(g(z))g'(z)^2 = \psi(z), \qquad \text{for all } z \in \Delta, g \in G,$$
$$\text{and} \quad \psi(z) = O(|z|^{-4}), \quad z \to \infty, \qquad \text{if } \infty \in \Delta$$

with the norm

$$\|\psi\|_\Delta = \sup_{z \in \Delta} \rho_\Delta(z)^{-2} |\psi(z)| ,$$

where $\rho_\Delta(z)|dz|$ is the Poincaré metric on the component of Δ containing z.

For every ϕ in $B_2(L, \Gamma)$, there exists a locally schlicht meromorphic function f_ϕ on L with $\{f_\phi, z\} = \phi(z)$; here $\{f, \cdot\}$ means the *Schwarzian derivative* of f

$$\{f, \cdot\} = (f''/f')' - (f''/f')^2/2 .$$

Throughout this paper, we shall denote by W_ϕ ($\phi \in B_2(L, \Gamma)$) a locally schlicht meromorphic function on L which is uniquely determined by ϕ such that

$$\{W_\phi, z\} = \phi(z)$$

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