J. Math. Kyoto Univ. (JMKYAZ)
28-1 (1988) 165-172

On well-posedness of the Cauchy problem for p-parabolic systems, II

By

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§1. Introduction.

Let $A(x,D)$ be a matrix of pseudo-differential operator of order p in the form

$$A(x, D)=H(x, D)\Lambda^{p}+B(x, D), \qquad x\in R^{l}, \tag{1.1}$$

where $H(x, \xi)$ is $m\times m$ homogeneous matrix of degree 0 in $\xi(|\xi|\geqq 1)$ and smooth in x and ξ. $B(x, \xi)$ belongs to the class $S^{p_0}_{1,0}$, $0\leqq p_0<p$, modulo smoothing operators. Here, the symbol of Λ belongs to $S^{1}_{1,0}$ (see for example, H. Kumano-go [2]) and coincides with $|\xi|$ for $|\xi|\geqq 1$ and p is a positive number.

The purpose of this paper is to show that the condition

$$\sup_{x\in R^{l},\ \xi\in S^{l-1}_{\xi}} \mathrm{Re}\ \lambda_i(x, \xi)<0, \qquad 1\leqq {}^{\forall}i\leqq m \tag{1.2}$$

is necessary and sufficient in order that there exist positive constants a, b and β such that the estimate

$$\|(\lambda I-A(x, D))U(x)\|\geqq a(|\lambda|-\beta_0)\|U\|+b\|U\|_p, \qquad \text{for } {}^{\forall}U\in H^{p},\ {}^{\forall}\lambda,\ \mathrm{Re}\ \lambda\geqq\beta_0 \tag{1.3}$$

holds.

Here $U(x)$ is m-vector, $\|.\|$, $\|.\|_p$ denote L^2 and H^p-norm respectively. $\lambda_i(x, \xi)$, $(i=1, 2,\dots m)$ are the roots of the characteristic equation

$$\det\ (\lambda I-H(x, \xi))=0.$$

Note that the sufficiency was proved in [1] by using a partition of unity of the unit sphere S^{l-1}_{ξ} and a partition of unity in R^{l}_{x} as in Mizohata [3]. Therefore, we need only to show the necessity of the condition (1.2).

In this article we shall use the method of micro-localization of pseudo-differential operators which was developed by Mizohata [4] and [5]. In §2. we give the definition of micro-localizer and state our result. In §3. we give the proofs of the proposition 2.1 and lemma 2.1.

Received December 18, 1986