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A comparison theorem for Riccati equations and its application to simple hyperbolic boundary value problems

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§1. Introduction.

In this paper we deal with a certain comparison theorem for Riccati equations and apply it to construct an integral representation of forward progressing solutions of simple hyperbolic boundary value problems. This integral representation enables us to see that the forward progressing solution preserves its order of singularities along characteristic curves. (See (P.1) and (P.2) below.)

Let us explain our comparison theorem (Theorem 2.1). Let $q(x)$ be a smooth function defined on $[0, \infty)$ taking its values in $C-(-\infty, 0]$. By $\sqrt{q(x)}$ we denote a root of $q(x)$ with positive real part. It is evident that, if $q(x)$ is constant q, the equation $w'=q-w^2$ has solutions $\sqrt{q}$ and $-\sqrt{q}$. Here $-\sqrt{q}$ is an unstable solution, $\sqrt{q}$ being stable. In general case it is interesting to seek for the solution $w(x)$ of $w'=q(x)-w^2$, which stays close to $-\sqrt{q(x)}$ for all $x\in[0, \infty)$. We suppose that $(\log q(x))'$ is not so large as compared with Re $\sqrt{q(x)}$. More precisely under the assumption

$$(1.1)\qquad D(q)=\sup_{0\leq x}\left\{\left|\frac{q'(x)}{q(x)}\right| \,/\, \mathrm{Re}\sqrt{q(x)}\right\}<4,$$

we can show that there exists a solution $w(x)$ of $w'=q(x)-w^2$ satisfying

$$(1.2)\qquad \left|\frac{w(x)-(-\sqrt{q(x)})}{w(x)-\sqrt{q(x)}}\right|<r_1, \text{ for all } x\in[0, \infty),$$

where r_1 stands for a root of $r+\frac{1}{r}=\frac{8}{D(q)}$ less than 1, i.e.

$$(1.3)\qquad r_1+\frac{1}{r_1}=\frac{8}{D(q)},\quad r_1<\frac{D(q)}{4}<1.$$

In order to apply the above result to the boundary problem stated below, we take $q(x)=-\tau^2 a(x)$, where $a(x)$ is a positive valued smooth bounded function and $\tau=\sigma-i\gamma$, $\sigma\in R$, $\gamma>0$. Then $D(q)$ can be made arbitrarily small if we take the

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