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Differentiable vectors and analytic vectors in completions of certain representation spaces of a Kac-Moody algebra

By

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Introduction.

Let $\mathfrak{g}_{\boldsymbol{R}}$ be a Kac-Moody algebra over the real number field $\boldsymbol{R}$ with a symmetrizable generalized Cartan matrix (GCM), and $\mathfrak{h}_{\boldsymbol{R}}$ the Cartan subalgebra of $\mathfrak{g}_{\boldsymbol{R}}$. Then, the Kac-Moody algebra $\mathfrak{g}$ over $\boldsymbol{C}$ corresponding to the same GCM and its Cartan subalgebra $\mathfrak{h}$ are given by

$$\mathfrak{g} = \boldsymbol{C}\otimes_{\boldsymbol{R}}\mathfrak{g}_{\boldsymbol{R}} \quad \text{and} \quad \mathfrak{h} = \boldsymbol{C}\otimes_{\boldsymbol{R}}\mathfrak{h}_{\boldsymbol{R}},$$

respectively. We denote by $\mathfrak{k}$ the unitary form of $\mathfrak{g}$, and put $\mathfrak{k}_{\boldsymbol{R}}=\mathfrak{k}\cap\mathfrak{g}_{\boldsymbol{R}}$ (for the precise definition, see [8] and [7]).

In [8] and [7], we constructed and studied groups K^{Λ} and $K^{\Lambda}_{\boldsymbol{R}}$ consisting of unitary operators on a Hilbert space $H(\Lambda)$ which is a completion of the integrable highest weight module $L(\Lambda)$ for $\mathfrak{g}$ with dominant integral highest weight $\Lambda\in\mathfrak{h}^{*}_{\boldsymbol{R}}$. These groups are generated by naturally defined exponentials of elements in $\mathfrak{k}$ and $\mathfrak{k}_{\boldsymbol{R}}$ respectively. In this paper, we show that the exponential map $\exp\colon \mathfrak{k}\rightarrow \boldsymbol{U}(H(\Lambda))$ can be extended to a certain completion $H^{u}_{1}(\text{ad})$ of $\mathfrak{k}$. We show, in prallel, that taking the adjoint representation of $\mathfrak{g}$ on itself in place of the highest weight representation on $L(\Lambda)$, and completing the representation space $\mathfrak{g}$ to a Hilbert space $H(\text{ad})$, the exaponintial map $\exp\colon H^{u}_{1}(\text{ad})\mapsto \boldsymbol{B}(H(\text{ad}))$ can be defined naturally. Here $\boldsymbol{U}(H)$ is the group of unitary operators and $\boldsymbol{B}(H)$ is the algebra of bounded operators on a Hilbert space H. Note that the adjoint representation is quite different from $L(\Lambda)$ at the point that the set of its weights is unbounded both in positive and negative directions when $\mathfrak{g}$ is of infinite-dimension. For these exponentials, we define the differentiable vectors and the analytic vectors, and prove some properties of them, which we expect to utilize for studying fine structures of K^{Λ} and $K^{\Lambda}_{\boldsymbol{R}}$.

Let us explain in more detail. We denote by $\underline{\mathfrak{g}}$ the infinite direct products of $\mathfrak{g}^{0}=\mathfrak{h}$ and the root spaces $\mathfrak{g}^{\alpha}$ over α, and by $\underline{L}(\Lambda)$ that of all the weight spaces $L(\Lambda)_{\mu}$ over μ, respectively. $\mathfrak{g}$ acts on $\underline{\mathfrak{g}}$ and $\underline{L}(\Lambda)$ naturally. Let $H(\text{ad})$ and $H(\Lambda)$ be the

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