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On extensions of higher derivations for algebraic extensions of fields of positive characteristics

Dedicated to Professor M. Nagata on his sixtieth birthday

By

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§0. Introduction

In the present paper, we will present systematical arguments on extensions of higher derivations for algebraic field extensions of positive characteristics. Arguments on ordinary derivations are included as special cases.

Assume that a higher derivation d is given in a field K of positive characteristic. Let L be an algebraic extension of K. If d is extended to higher derivations of L, we denote one of them by d'.

In §2, we make basic considerations on relationship among constant fields and value domains, of d and d'. In §3, we seek conditions that d can be extended to higher derivations of L. In the case where d is an iterative higher derivation of finite rank we get a conclusion successfully and then we get a criteria for L to be maximal in the set of algebraic extensions of K to which d can be extended. In §4 and §5, after we discuss conditions that the extension of d is unique and conditions that the extension of d keeps the property of being iterative when d is iterative, we show that in the case where d is a higher derivation of infinite rank, there exists the largest algebraic extension of K to which d can be extended. Finally in §6, we discuss non-integrable elements. Actually, the corollary to Theorem 6-1 about this matter for ordinary derivations, has given the author a motivation to start this work. The author has tried to find a literature in which it is explicitly stated. But he has not been able to find one so far, except for that R. Baer in his paper [1] touched upon it under some restricted conditions. We conclude §6 in proving that if a higher derivation d of K is iterative and of infinite rank, then each non-integrable element of an arbitrary order in K is non-inrtegrable for every extension d' of d on an algebraic extension of K, as long as the index of d' equals the index of d. (For definitions of a non-integrable element and the index, see §1.) This may correspond to the fact that an integration of a rationally non-integrable element is transcendental, in the case of characteristic 0.

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