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Embeddings of discrete series into induced representations of semisimple Lie groups, II

—Generalized Whittaker models for $SU(2, 2)$—

By

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Introduction

This is the second part of our work on embeddings of discrete series into various, important induced modules for semisimple Lie groups. Applying the general method established in the first part [10] (referred as [I] later on), we describe in this paper (generalized) Whittaker models for the simple Lie group $SU(2, 2)$ in an explicit way.

To be precise, we consider representations smoothly induced from characters of the unipotent radical of a cuspidal parabolic subgroup. The infinitesimal embeddings of discrete series are determined almost completely for such induced modules. Among other things, through this series of works we find all the embeddings into Gelfand-Graev representations, and also the zero-th $\mathfrak{n}$-cohomologies for the discrete series of $SU(2, 2)$. Note that our group is of real rank two, and that it is locally isomorphic to the (restricted) conformal group on the Minkowski space.

Now, let G be a connected semisimple Lie group with finite center, and K a maximal compact subgroup of G. We always assume the rank condition: $\mathrm{rank}(G)=\mathrm{rank}(K)$, which is necessary and sufficient for G to have a non-empty discrete series [2]. Each discrete series π_Λ of G has a unique lowest K-type τ_λ with highest weight λ(see 1.1). Further, the representation π_Λ can be realized on the L^2-kernel of gradient-type, G-invariant differential operator D_λ(see [7], cf. [I, Th. 1.5]). This D_λ is defined on the G-vector bundle over $K\backslash G$ attached to the K-module τ_λ.

From this realization of π_Λ, we can deduce that the L^2-kernel of D_λ characterizes the embeddings of π_Λ^*, the contragredient of π_Λ, into the left regular representation of G on $L^2(G)$. In fact, the exterior tensor product $\pi_\Lambda^*\hat{\otimes}\pi_\Lambda$ occurs in the bi-regular representation of $G\times G$ just once, and the functions in L^2-$\mathrm{Ker}(D_\lambda)$ give rise to lowest K-type vectors in $L^2(G)$ of type $\tau_\lambda^*\subset\pi_\Lambda^*|K$ with respect to the left K-action.

Suggested by this fact, we formulated in the first half of [I] a general method for describing infinitesimal embeddings of discrete series into C^∞-induced G-modules. This is done by letting the operator D_λ act on the τ_λ^*-component of the induced module $\pi(\eta)=C^\infty\text{-}\mathrm{Ind}_N^G(\eta)$ mentioned above, in a natural way (see 1.3 for the the precise definition). We have shown that, as in the regular representation case, solutions φ of the

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