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Absence of the affine lines on the homology planes of general type

By

M. MIYANISHI AND S. TSUNODA

Introduction

Let X be a nonsingular algebraic surface defined over the complex field $\mathbf{C}$. We call X a *homology plane* (resp. $\mathbf{Q}$-*homology plane*) if the homology groups $H_i(X; \mathbf{Z})$ (resp. $H_i(X; \mathbf{Q})$) vanish for all $i > 0$. A purpose of the present article is to show the following result.

Main Theorem. *Let X be a $\mathbf{Q}$-homology plane of Kodaira dimension* 2. *Then there lies no curve C on X which is topologically isomorphic to the affine line $\mathbf{A}^1$.*

The core of a proof is to show that X and $X - C$ are respectively embedded as Zariski open sets into almost minimal pairs (cf. [9]; see below) and that the inequality of Miyaoka-Yau type (cf. [5], [10]), after a relevant modification, can be applied to derive a contradiction if one assumes the existence of a curve topologically isomorphic to $\mathbf{A}^1$.

M. Zaidenberg [11] informed us of the following theorem which overlaps our main theorem and whose proof is to be published in Math. USSR, Izvestija.

Theorem of Zaidenberg. *Let X be a homology plane which is not isomorphic to $\mathbf{A}^2$. Then the following conditions are equivalent to each other*:

(1) *There exists a curve Γ_0 in X which is isomorphic to $\mathbf{A}^1$*;

(2) *There exists a simply connected curve Γ_0 in X which is* a posteriori *isomorphic to $\mathbf{A}^1$*;

(3) *There exists an isotrivial family of curves $X \to C$, which is not a singular $\mathbf{C}^{**}$-family*;

(4) *There exists a regular map $X \to \mathbf{P}^1$ with $\mathbf{C}^*$ as a general fiber*;

(5) *X has Kodaira dimension* 1.

1. Almost minimal surfaces and inequalities of Miyaoka-Yau type

Let (V, D) be a pair consisting of a nonsingular projective surface V and a reduced effective divisor D with simple normal crossings. Denote by K_V the

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