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On the coordinate-free description of the conformal blocks

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Introduction

Recent study of conformal field theory has provided new insights and techniques concerning the moduli space of stable curves. Among them is an extensive study on an interplay between the representation theory of infinite-dimensional Lie algebras and the theory of vector bundles on the moduli space of stable curves. The basic idea is easy. Consider the moduli space of framed stable curves (stable curves with coordinates). While it is clear that the Lie algebra of infinitesimal change of coordinates acts on this moduli space, interesting point is that even *singular* change of coordinates acts on this moduli space. The effect of singular coordinate change is to deform the shape of curves. Thus the Lie algebra of infinitesimal changes of coordinates, possibly with singularities, serves as a Lie algebra of infinitesimal symmetry of the moduli space. The algebra plays an important role in the conformal field theory. In certain circumstances, however, the universal central extension of the algebra, called the Virasoro Lie algebra, is more convenient to use. It gives a symmetry of the determinant line bundle on the moduli space. It is also possible to consider the algebra of infinitesimal change of trivialization, possibly with singularities, as an algebra of infinitesimal symmetry of the moduli space of framed vector bundles. The central extension of this algebra is called an affine Lie algebra. Similar to the theory of vector bundles on a symmetric space, an interplay of Lie algebra representation and vector bundles on the moduli space thus arises.

Following this idea, Tsuchiya, Ueno, and Yamada [TUY] constructed vector bundles, called sheaves of vacua, over base schemes of local universal families of framed stable curves. They further showed that there is a projective connection on each such vector bundle. But we may ask what kind of a role the use of coordinates plays. We want a coordinate-free description of the theory to study the mechanism of this procedure deeper. In fact, it is proved in [TUY] that the choice of coordinate is unessential, but it is not at all clear a priori. In this paper we give a simple and coordinate-free description of the construction given there, and clarify the nature of the theory. In the way we deal with the notion of normal ordering, which is used in the construction of the projective

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