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Nondiscrete local ramified class field theory

By

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1. Introduction

Let p be a prime, Q_p the field of p-adic numbers, Ω an algebraic closure of Q_p and $\bar{\Omega}$ the (topologic) completion of Ω. Suppose k is an infinite algebraic extension of Q_p with finite residue field and such that the exponent of p in the Steinitz number $[k: Q_p]$ is finite, and $\bar{k}$ its (topological) completion. We study the finite abelian totally ramified extensions of k and $\bar{k}$, in terms of subgroups of norms of $U(k)$ and $U(\bar{k})$ respectively. More precisely, if ℓ is a finite abelian extension of k and $\bar{\ell}$ its completion, then one has the following commutative diagram

$$\begin{array}{ccc} U(\bar{k})/\bar{H} & \xrightarrow{\delta_{\bar{\ell}/\bar{k}}} & \mathrm{Gal}\,(\bar{\ell}/\bar{k})_{\mathrm{ram}} \\ {\scriptstyle\phi}\uparrow & & \uparrow{\scriptstyle\mathrm{Res}^{-1}} \\ U(k)/H & \xrightarrow{\delta_{\ell/k}} & \mathrm{Gal}\,(\ell/k)_{\mathrm{ram}} \end{array}$$

where all the arrows are functorial isomorphisms, and H and $\bar{H}$ are the subgroups of norms of units from ℓ and $\bar{\ell}$ respectively. Moreover, one has a continuous group homomorphism

$$\widehat{U(k)} \xrightarrow{\delta_k} \mathrm{Gal}\,(k_{\mathrm{ab}}/k)_{\mathrm{ram}}$$

(where $\widehat{U(k)}$ is the completion of $U(k)$ with respect to the subgroups of finite index), which is surjective and whose kernel is the subgroup of roots of unity in $U(k)$ of order $q_1 = (q-1, [k: Q_p]_\infty)$.

Throughout the paper use ideas and results of Hazewinkel's ([3]) and Iwasawa's ([5]).

As a remark, here we describe the finite abelian extensions (totally ramified) of an infinite totally ramified extension of a local field with only finite wild ramification, while J.M. Fontaine and J.P. Wintenberger do it for totally ramified extensions of a local field with only finite tame ramification ([2]). Our next goal is to put these two together.

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