J. Math. Kyoto Univ. (JMKYAZ)
37-1 (1997) 129-150

High order Itô-Taylor approximations to heat kernels*

By

Arturo KOHATSU-HIGA

1. Introduction

Let $(\Omega, \mathcal{F}, P)$ be the canonical space of the standard m dimensional Wiener process $W=(W^1, \cdots, W^m)$. On this space, define X as the solution of the following SDE (stochastic differential equation):

$$X_t = x + \sum_{i=1}^{m} \int_0^t B_i(X_s) dW_s^i + \int_0^t B_0(X_s) ds, \quad 0 \leq t \leq T, \tag{1.1}$$

where, $x \in \mathbf{R}^d$ and B_i: $\mathbf{R}^d \to \mathbf{R}^d$, $i=0, \cdots, m$ are smooth vector fields with bounded derivatives. It is known that under the condition that

$$(\det \sigma_{X_t})^{-1} \in \bigcap_{p \geq 1} L^p(\Omega), \tag{1.2}$$

X_t has a smooth density, say $q(t; x, y)$. Here σ_F denotes the Malliavin covariance matrix of the random variable F.

The purpose of this article is to find ways of approximating $q(t; x, y)$. Recently, Hu-Watanabe [4] and Bally-Talay [1], have obtained results on this problem. Let's introduce these results. Define the following sets of vector fields

$$\Sigma_0 = \{B_j, j=1, \cdots, m\}$$
$$\Sigma_j = \{[B_k, V]; V \in \Sigma_{j-1}, k=0, \cdots, m\}, j \geq 1,$$

where $[\cdot, \cdot]$ denotes the Lie bracket. Now define for $A \geq 1$, the quadratic forms

$$V_A(x, \eta) := \sum_{V \in \Sigma_{A-1}} \langle V(x), \eta \rangle^2$$

and set

$$V_A(x) = 1 \wedge \inf_{|\eta|=1} V_A(x, \eta). \tag{1.3}$$

Communicated by Prof. S. Watanabe, May 20, 1996

* This article was written while the author was visiting the Department of Mathematics at Kyoto University with a JSPS fellowship