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Chern classes for parabolic bundles

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1. Introduction

This is a continuation of our earlier works, [Bi2], [Bi3], where we studied various properties of parabolic bundles (both on curves and on higher dimensional varieties). Parabolic bundles (introduced in [MS] for curves and generalized to higher dimension in [MY]) are vector bundles (or more generally torsion-free coherent sheaves) on open varieties together with a weighted filtration at the boundary. Various results on vector bundles over projective manifolds generalize to the parabolic context.

Here we give a candidate for what should be the Chern classes of a parabolic bundle. Taking a hint from the definition of parabolic degree, which should be the first parabolic Chern class, of a parabolic bundle, in Section 3 we define parabolic Chern classes. (Indeed, the definition of the parabolic degree in higher dimensions, which is rather nontrivial (introduced in [MY]), serves as a good hint.)

Given a representation in $GL(r, \mathbf{C})$ of the fundamental group of a smooth open variety, there is a natural extension of the corresponding flat bundle to some suitable compactification (the divisor at infinity should be of normal crossing) as a parabolic bundle. We give a justification for our definition of parabolic Chern classes by pointing out that all the parabolic Chern classes of such a parabolic bundle vanish.

S. Bloch and D. Gieseker showed that the Chern classes of an ample vector bundle are numerically positive. This result was extended in [FL], and all the numerically positive characteristic polynomials for ample vector bundles were identified. In [Bi2] we defined parabolic ample bundles and showed that they exhibit various properties analogous to an ample vector bundle—for example, Hartshorne's characterization of ample vector bundles on curves, Le Potier vanishing theorem.

In Section 4a we show that the parabolic Chern classes of a parabolic ample bundle are numerically positive. The statement correpponding to the theorem of [FL] is also valid. We prove that under certain conditions on the filtration, a parabolic stable bundle with vanishing parabolic Chern classes share the characteristics of a stable vector bundle with vanishing Chern

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