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Meyer's inequality of the type $L(\log L)^{\alpha}$ on abstract Wiener spaces

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1. Introduction

Let (W, H, μ) be an abstract Wiener space, that is,

(i) W is a real separable Banach space,

(ii) H is a real separable Hilbert space which is continuously and densely imbedded in W,

(iii) μ is a Borel probability measure on W such that

$$\int_W \exp\left(\sqrt{-1}(f, w)\right) d\mu(w) = \exp\left(-\frac{1}{2}\|f\|_{H^*}^2\right)$$

for any $f \in W^*$. Here we identify H^* with H by the Riesz theorem so that $W^* \hookrightarrow H^* = H$. As usual, we denote by D H-derivative and by L be the Ornstein-Uhlenbeck operator, cf. [13].

As for the continuity of operators D and L with respect to L^p-norms on the Wiener space, the following inequality due to Meyer [8] is well-known: for $1 < p < \infty$,

$$\|D\phi\|_p \lesssim \|(I-L)^{1/2}\phi\|_p \lesssim \|D\phi\|_p + \|\phi\|_p. \tag{1.1}$$

Here $A(\phi) \lesssim B(\phi)$ means that there exists a constant $k > 0$ independent of ϕ such that $A(\phi) \leq kB(\phi)$ holds for every ϕ. (1.1) implies in particular that the operator $D(I-L)^{-1/2}$ is continuous from $L^p(\mu)$ to $L^p(\mu : H)$ (L^p-space with values in $H^* = H$) if $1 < p < \infty$. However this continuity fails when $p = 1$ and the main purpose of this article is to study the continuity of the operator $D(I-L)^{-1/2}$ with respect to the $L(\log L)^{\alpha}$-topology.

Meyer's proof of (1.1) relies on the Littlewood-Paley inequality. Recently, a simplified proof was given by Pisier [9], or Feyel [3], by using the L^p-continuity of the Hilbert transform on the circle. L^1-continuity of the Hilbert transform no longer holds. However, it is continuous as an operator from $L \log L$ to L^1 (cf. [6]). Taking this fact into account, a natural question arises: Can we show that

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