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The Steenrod algebra and the automorphism group of additive formal group law

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1. Introduction

Let H_*H be the Hopf algebra of stable co-operations of the mod 2 ordinary cohomology theory $H^*(\ )$. The structure of H_*H is well known as follows. First J. P. Serre [7] determined the unstable cohomology of the Eilenberg-MacLane complex $K(n,\mathbb{Z}/2)$. He has shown the stable part of $H^*(K(n,\mathbb{Z}/2))$ is generated by iterated Steenrod operations and computed the rank of $H^i(K(n,\mathbb{Z}/2))$ in terms of excess operations. He assumed the existence of Steenrod squares Sq^i but did not use the Adem relations. Using the Adem relations, we see that the algebra S^* generated by Steenrod squares modulo the Adem relations is isomorphic to H^*H. Moreover Milnor [4] determined the Hopf algebra structure of S_*, the dual Steenrod algebra which is the polynomial algebra $\mathbb{F}_2[\xi_1,\xi_2,\dots]$ with the coproduct $\psi(\xi_n)=\sum_{i=0}^{n}\xi_{n-i}^{2^i}\otimes\xi_i$, and therefore we obtain the Hopf algebra structure of H_*H.

Now we recall strict automorphisms of the additive formal group law. Let G_a be the additive formal group law, and $\mathrm{Aut}_{\mathbb{F}_2}(G_a)(R_*)$ the set of strict automorphisms of G_a over a non-negatively graded commutative $\mathbb{F}_2$-algebra R_*. An element $f(x)$ in $\mathrm{Aut}_{\mathbb{F}_2}(G_a)(R_*)$ is written as a formal power series $x+\sum_{i=1}^{\infty}a_ix^{2^i}$, where $a_i\in R_{2^i-1}$. Here $\mathrm{Aut}_{\mathbb{F}_2}(G_a)(-)$ is a functor from the category of graded algebras to the category of sets. A product of $\mathrm{Aut}_{\mathbb{F}_2}(G_a)(R_*)$ is defined by the composition of power series, and induces the group structure. Therefore $\mathrm{Aut}_{\mathbb{F}_2}(G_a)(-)$ is a functor to the category of groups, and is represented by the Hopf algebra $A_*=\mathbb{F}_2[\bar\xi_1,\bar\xi_2,\dots]$ with the coproduct $\psi(\bar\xi_n)=\sum_{i=0}^{n}\bar\xi_{n-i}^{2^i}\otimes\bar\xi_i$. In other words, we have a natural group isomorphism

$$\mathrm{Hom}_{\mathbb{F}_2\text{-alg}}(A_*,R_*)\cong\mathrm{Aut}_{\mathbb{F}_2}(G_a)(R_*).$$

Comparing S_* with A_*, we see that $S_*\cong A_*$ as a Hopf algebra.

We recall the Dickson algebra. Let V^n be the $\mathbb{F}_2$-vector space spanned by elements $x_1,\dots,x_n$. In the polynomial ring $\mathbb{F}_2[x_1,\dots,x_n][t]$, consider the

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