

SOME SOFIC SHIFTS CANNOT COMMUTE WITH NONWANDERING SHIFTS OF FINITE TYPE

MIKE BOYLE

ABSTRACT. Suppose S is a nonwandering shift of finite type (SFT), and T is an expansive automorphism of S . We show T cannot be a strictly sofic almost Markov shift. Also included is an example of D. Fiebig, a reducible SFT with an expansive automorphism which is not SFT.

1. Introduction

In his 1995 memoir [19], Masakazu Nasu asked if an expansive automorphism of a shift of finite type must be a shift of finite type ([19, p.33, Question 2.a]). As already announced in [3], Doris Fiebig produced a counterexample in the case that the shift of finite type is allowed to be reducible. Her example is included in the Appendix.

The irreducible case of Nasu's question remains a major open problem for understanding the dynamics of automorphisms of shifts of finite type and the related \mathbb{Z}^d actions. There has been excellent progress on related questions involving positively expansive maps or onesided shifts of finite type [3], [7], [19], [20], [21]; however, since D. Fiebig's counterexample, there have been no results on Nasu's original question, despite considerable efforts.

In this paper, we will at least resolve a meaningful case of Nasu's question. We will prove that an expansive automorphism of a nonwandering shift of finite type cannot be conjugate to an almost Markov strictly sofic shift. In the example of D. Fiebig, the expansive automorphism of the reducible shift of finite type is almost Markov (Remark A.4), and the reducible shift of finite type is wandering. The two results together highlight the importance of the irreducibility condition, or more precisely the condition that the SFT be nonwandering (Remark 3.4).

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