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NOTE ON H^1 SPACES RELATED TO DEGENERATE SCHRÖDINGER OPERATORS

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ABSTRACT. Let $\mathcal{L}f(x) = -\frac{1}{w(x)}\sum_{i,j}\partial_i(a_{ij}(\cdot)\partial_j f)(x) + V(x)f(x)$, where w is a weight from the Muckenhoupt class A_2, V is a nonnegative potential that belongs to a certain reverse Hölder class with respect to the measure $w(x)\,dx$, and $C^{-1}w(x)|\xi|^2 \le \sum_{i,j} a_{ij}(x)\xi_i\bar{\xi}_j \le Cw(x)|\xi|^2$. Let $\{T_t\}_{t>0}$ be the semigroup of linear operators generated by $-\mathcal{L}$. We say that a function f is an element of the space $H^1_{\mathcal{L}}$ if the maximal operator $\mathcal{M}f(x) = \sup_{t>0}|T_t f(x)|$ belongs to $L^1(R^d(w(x)\,dx))$. A special atomic decomposition of $H^1_{\mathcal{L}}$ is proved.

1. Introduction

On $\mathbb{R}^d$ we consider a degenerate Schrödinger operator $\mathcal{L}$ having the form

$$\mathcal{L}f(x) = -\frac{1}{w(x)}\sum_{i,j}\partial_i(a_{ij}(\cdot)\partial_j f)(x) + V(x)f(x), \tag{1.1}$$

where $a_{ij}(x)$ is a real symmetric matrix satisfying

$$C^{-1}w(x)|\xi|^2 \le \sum_{i,j} a_{ij}(x)\xi_i\bar{\xi}_j \le Cw(x)|\xi|^2, \tag{1.2}$$

with w being a nonnegative weight from the Muckenhoupt class A_2, and $V \ge 0$ belonging to a reverse Hölder class with respect to the measure $d\mu(x) = w(x)\,dx$ (cf. (2.5)). Denote by $\mathcal{E}(f,g)$ the Dirichlet form associated with $\mathcal{L}$, that is,

$$\mathcal{E}(f,g) = \int_{\mathbb{R}^d}\sum_{i,j} a_{ij}(x)\partial_j f(x)\overline{\partial_i g(x)}\,dx + \int_{\mathbb{R}^d} V(x)f(x)\overline{g(x)}\,d\mu(x).$$

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