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ADDENDUM TO "A FIXED POINT THEOREM FOR BOUNDED DYNAMICAL SYSTEMS"

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We say that a compact set W is a *window* for a dynamical system (continuous map or flow) on X if the forward orbit of every point $x \in X$ intersects W. If a dynamical system has a window then we will say that it is *bounded*.

In [6], we proved the following fixed point theorem.

THEOREM 1. *Every bounded dynamical system on $\mathbb{R}^n$ has a fixed point.*

Since that paper appeared, it has come to our attention that Theorem 1 had already been proved, in [2] in the case of maps and [9] in the case of flows. We wish to correct our oversight in this addendum.

Fournier proved the following:

THEOREM 2 ([2]). *If X is an absolute neighborhood retract, $f : X \to X$ is a locally compact, bounded map, and the Lefschetz number of f is nonzero, then f has a fixed point.*

(A map is locally compact if every point has a neighborhood whose image is precompact.) Theorem 1 for maps is thus a special case of this result. In fact, the results in [2] go far beyond Theorem 2 and are concerned with finding very general circumstances in which the Lefschetz fixed point theorem applies. Further important work in this area appears in [1], [3], [4].

Srzednicki proved the following result for flows:

THEOREM 3 ([9]). *If X is a Euclidean neighborhood retract and φ is a bounded flow on X, then X has the homotopy type of a compact polytope, and φ has a fixed point provided that the Euler characteristic of X is nonzero.*

Theorem 1 for flows is a special case of this result. Further results on bounded flows appear in [5].

Theorem 1, being less general than these two theorems, has at least the virtue of a much shorter proof. In addition, the outlook is rather different. The

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