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A GAUSSIAN AVERAGE PROPERTY OF BANACH SPACES

P.G. CASAZZA AND N.J. NIELSEN

Introduction

In this paper we introduce a Gaussian average property, abbreviated *GAP*. A Banach space X is said to have *GAP* if there is a constant K so that $\ell(T) \leq K\pi_1(T^*)$ for every finite rank operator from ℓ_2 to X. Here $\ell(T)$ denotes the ℓ-norm defined by Linde and Pietsch [7]; see also N. Tomczak-Jaegermann [13].

We investigate this property in detail and establish that a large class of Banach spaces have it. It turns out that every Banach space which is either of type 2 or is isomorphic to a subspace of a Banach lattice of finite cotype has *GAP* and so does a Banach space of finite cotype which has the Gordon-Lewis property GL_2 with respect to Hilbert spaces.

GAP and GL_2 are closely related, and this enables us to obtain some results on GL_2 by investigating *GAP*. We prove for example, that *GAP* and GL_2 are equivalent properties for cotype 2 spaces and that a K-convex Banach space X has GL_2 if and only if both X and X^* have *GAP*. It also turns out that if a space X is of finite cotype and X^* has *GAP*, then X is K-convex.

We also prove that *GAP* gives rise to some extension theorems of operators with range in a Hilbert space. We prove for example, that if X has *GAP*, then every operator from a subspace of X into a Hilbert space, which factors through L_1, extends to an L_1-factorable operator defined on X. Further, if the dual of a subspace E of a finite cotype Banach space X has *GAP*, then every absolutely summing operator from E to a Hilbert space extends to an absolutely summing operator defined on X. If X^* has *GAP* then the other direction is true for all subspaces E of X. This implies that if X is a Banach space of finite cotype with GL_2 then a subspace E has GL_2 if and only if every 1-summing operator from E to a Hilbert space extends to a 1-summing operator defined on X.

We now wish to discuss the arrangement and contents of the paper in greater detail.

In Section 1 we prove the major results on *GAP* mentioned above. One of the main tools for obtaining these is the duality theorem 1.7 which also relates *GAP* to K-convexity. We provide several examples of Banach spaces with a reasonable structure which fail *GAP*. At the end of the section it is shown that the ℓ_2-sum of

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