# THE VOLUME OF A TUBE IN COMPLEX PROJECTIVE SPACE

BY

F. J. FLAHERTY

## 1. Introduction

The relationship between the volume of a tube around a submanifold and integral invariants of the submanifold has interested geometers for years. Steiner considered the problem as long ago as 1840 [10].

Many results on the subject appear in the 1940's but Hermann Weyl's work in 1939 [11] yields the definitive answer for tubes around submanifolds in the model spaces of Riemannian geometry (euclidean, spherical, or hyperbolic space). In fact, Weyl's results are so powerful that the first general Gauss-Bonnet theorem, proved by Allendoerfer [1] and Allendoerfer-Weil [2], used Weyl's formula in a fundamental manner. Of course, all of this took place before Chern provided us with his intrinsic proof of the general Gauss-Bonnet theorem [3].

In this paper we compute the volume of a tube around a compact subdomain, with smooth boundary, of a holomorphic submanifold of complex projective space. Essentially, we identify certain extrinsically defined functions as intrinsic scalar densities. The computation appears in Section 4.

A very crude estimate of the sum of the Betti numbers of the path space of a submanifold of a pinched manifold appears in work of Flaherty and Grossman [6]. In fact, in the present paper, we prove that the sum of the first $\lambda$ Betti numbers of the path space of a compact holomorphic submanifold of complex projective space is dominated by a linear polynomial in $\lambda$.

Sections 2 and 3 serve as background for the main theorems, found in Sections 4 and 5. Section 2 recalls basic ideas and fixes notation for complex projective space necessary to the calculations in Section 4 while Section 3 reviews the local geometry of holomorphic submanifolds of Kaehler manifolds, on which the remainder of the paper rests.

We plan in the future to investigate the relation of this formula to equidistribution theory of holomorphic curves [13].

## 2. Complex projective space

We devote this section to the geometry of complex projective space, the ambient space for our submanifolds.

Let $\mathbf{C}^{n+1}$ be the space of $(n+1)$-tuples of complex numbers and $e_0 \cdots e_n$ a frame field on $\mathbf{C}^{n+1}$; then

$$de_A = \textstyle\sum_B e_B \omega_A^B \quad (0 \leq B \leq n) \quad \text{and} \quad d\omega_B^A = \textstyle\sum_C \omega_C^A \wedge \omega_B^C \quad (0 \leq C \leq n).$$

Received June 4, 1970.