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SPLITTING THEOREMS FOR QUADRATIC RING EXTENSIONS

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1. Introduction

Let R be a regular Noetherian ring (all rings are commutative, with identity) and let $S \supset R$ be a module-finite extension algebra. It is an open question whether $R \hookrightarrow S$ splits as a map of R-modules, i.e., whether the copy of R in S has an R-module complement E such that $S = R \oplus_R E$. This is known if R contains a field, and also if S_m has a big Cohen-Macaulay module for every maximal ideal m of S (see [2]). The question can be reduced to the case where S is a domain (see [2]).

We shall show here that when S is a domain such that the extension of fraction fields is quadratic the answer is affirmative: In fact, it suffices that R be supernormal and locally factorial, where "supernormal" means that the Serre conditions R_2 and S_3 hold (see [7, p. 124]). The main case is where R is of mixed characteristic 2.

Moreover, we give an interesting almost "generic" counterexample when the condition R_2 is weakened: In this example, the ring is a factorial *complete* local domain of mixed characteristic 2 which is a hypersurface. The most difficult feature of this example is to prove factoriality after completion: This is achieved by representing the hypersurface as a ring of invariants and calculating group cohomology (cf. [1], [2]).

It has recently been shown [6] that the direct summand conjecture has the same homological consequences (i.e., implies the same standard homological conjectures) as does the existence of big Cohen-Macaulay modules. This focuses increased attention on the direct summand conjecture. Further discussion of the conjectures may be found in [3], [4], [5], [6], [8], [9] and [11].

2. The Splitting Theorems

(2.1) THEOREM. *Let R be a locally factorial Noetherian domain which satisfies R_2 and S_3, e.g., a regular Noetherian domain, and let S be a*

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