ILLINOIS JOURNAL OF MATHEMATICS
Volume 31, Number 4, Winter 1987

# ARENS SEMI-REGULARITY OF THE ALGEBRA OF COMPACT OPERATORS

BY

MICHAEL GROSSER

## 1. Introduction

Quite a number of important Banach algebras occurring in functional analysis or harmonic analysis are not Arens regular, i.e., the two canonical extensions of the multiplication given on $A$ to the bidual $A^{**}$ are not identical. As J. Pym puts it in [18]: "In practice, regularity appears to be the exception rather than the rule." For example, the group algebra $L^1(G)$ of any infinite locally compact Hausdorff group $G$ or the algebra $K_0(X)$ of operators uniformly approximable by operators of finite rank, for a non-reflexive Banach space $X$, are not (Arens) regular [21], [22].

Among the Banach algebras possessing a two-sided bounded approximate identity, there is a subclass for which the Arens products still behave in a reasonable way although they need not to be identical. This class of so-called (Arens) semi-regular Banach algebras was introduced and described to some extent in [9]. As V. Losert and H. Rindler have shown, $L^1(G)$ is semi-regular if and only if $G$ is discrete or abelian [15]. Further, $K_0(X)$ is semi-regular if $X^*$, the dual of $X$, possesses the Radon-Nikodym property [9]. (When discussing semi-regularity of $K_0(X)$, $X^*$ always is assumed to possess the bounded approximation property so as to ensure that $K_0(X)$ contains a bounded approximate identity.)

These examples suggest that the notion of semi-regularity might be quite appropriate in the sense that it corresponds to important "classical" properties of the objects involved in applications.

It is the purpose of this paper to describe in some detail the relation between properties of $X$ and $X^*$ respectively (such as the Radon-Nikodym property) and semi-regularity of $K_0(X)$. Sections 1–5 contain the results valid for a general Banach space; in Section 6, the space $C(K)$ of continuous, complex-valued functions on a compact topological space $K$ and spaces $L^1(\mu)$ of equivalence classes of integrable functions are considered.

I am indebted to W. Schachermayer for valuable suggestions concerning the last section. The discussions we had provided the basis for the complete

Received January 7, 1985.