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THE INVARIANTS OF THE CONGRUENCE SUBGROUPS $G_0(\mathfrak{P})$ OF THE HECKE GROUP G_5

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1. Introduction

The modular group Γ together with its congruence subgroups have been extensively studied; the formulae for the geometric invariants of the principal congruence subgroups are well-known (see [5]) for example). One can define in an analogous way the principal congruence subgroups of the Hecke groups G_q $(q = 3, 4, 5, \dots)$ and ask for analogous formulae for the geometric invariants of these subgroups. Posed in this generality, this is not an easy question, for example, although it is easy to find upper bounds for the index of these subgroups, there are examples where these bounds are not attained and a general formula for the actual index is not known (see [2]). The purpose of this paper is to study the geometric invariants for the congruence subgroups of G_5 corresponding to the subgroups $\Gamma_0(N)$ of Γ.

Let

$$\lambda = 2\cos(\pi/5) = \frac{1+\sqrt{5}}{2},$$

and let G denote the Hecke group G_5, that is, the group of linear fractional transformations acting on the upper-half complex plane

$$\mathbf{H} = \{z \in \mathbf{C}\colon \operatorname{Im}(z) > 0\}$$

generated by the transformations

$$z \to -\frac{1}{z}, \quad \text{and} \quad z \to z + \lambda.$$

Thus, G may be identified with the subgroup

$$\left\langle \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \right\rangle \quad \text{of } \mathrm{PSL}_2(\mathbf{Z}[\lambda]).$$

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