ON AUTOMORPHIC FORMS OF NEGATIVE DIMENSION[1]

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1.[2] An automorphic form of positive dimension on an H-group is completely determined by its principal parts at the parabolic cusps; a form of zero dimension is determined up to an additive constant. The classical circle method of Hardy-Ramanujan-Rademacher-Zuckerman yields explicit expressions for the Fourier coefficients of forms of nonnegative dimension on the modular group and on certain of its subgroups. Recently we showed how this method could be modified to cover all H-groups, although for forms of zero dimension the Fourier coefficients are given only up to a bounded error term [1, Ch. IX], [2], [3], [4].

The situation is quite different when we consider automorphic forms of *negative* dimension. There may exist nonconstant forms of negative dimension that are regular everywhere including the cusps; in particular, there may exist *cusp* forms, that is, forms which vanish at the parabolic cusps. Hence in general the Fourier coefficients of a form of negative dimension can only be determined by its principal parts up to the order of magnitude of the Fourier coefficients of an everywhere regular form.

It is the purpose of this paper to show that the circle method suffices to determine the Fourier coefficients of forms of negative dimension also, insofar as these are determined by their principal parts. The circle method is thus revealed as a uniform method, valid for all dimensions, for extracting all possible information from the principal parts of an automorphic form. As we remark at the end of this section, the same statement holds for automorphic *integrals*.

Let

$$\begin{aligned} g(m, r) &= m^{r/2} && \text{if} \quad 0 < r < 2, \\ &= m \log m && \text{if} \quad r = 2, \\ &= m^{r-1} && \text{if} \quad r > 2. \end{aligned} \tag{1}$$

If $c_m^{(k)}$ is the m^{th} Fourier coefficient of an everywhere regular form $G(\tau) \in \{\Gamma, -r, v\}$, i.e., of dimension $-r$ and multiplier v, it is known that ([5], cf. also [6])

$$c_m^{(k)} = O(g(m, r)). \tag{2}$$

For $r > 2$ this estimate is best possible, as the Eisenstein series show. At any rate (2) is the best result presently obtainable for all H-groups by the

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[2] For definitions and notation, cf. [1, Chapters VIII, IX]; also Section 2, below.