WEAK CONVERGENCE OF PROBABILITIES ON NONSEPARABLE METRIC SPACES AND EMPIRICAL MEASURES ON EUCLIDEAN SPACES

BY

R. M. DUDLEY[1]

1. Introduction

It is known that under certain mild set-theoretic assumptions, a finite, countably additive measure defined on all Borel sets of a metric space is concentrated in a separable subspace (Marczewski and Sikorski [8]). However, there are interesting probability measures on metric spaces not concentrated in separable subspaces. In this paper, we consider countably additive probability measures on the smallest σ-field containing the open balls of a metric space. This σ-field is the Borel field for a separable space, but is smaller in general. A probability measure on it need not be confined to a separable subspace.

A sequence of such measures will be said to converge weak* to a Borel measure if the upper and lower integrals of each bounded continuous real function converge. Some abstract results on this convergence, similar to those in Prokhorov [9] for separable metric spaces, will be given in §2.

The rest of the paper deals with "empirical measures" on Euclidean spaces, whose study motivated the abstract results and provides an application of them. Two of the main results of Donsker [3], [4] for measures on the real line will be generalized to arbitrary Euclidean spaces. At the same time, his results are corrected by replacing some integrals, which may not be defined, by upper and lower integrals.

I discovered after writing most of the rest of this paper that a generalization of Donsker's work to multidimensional spaces was proved several years ago by L. LeCam, who is now revising a paper embodying his results for the Illinois Journal of Mathematics. I shall try to explain what seem to be the main differences between our approaches.

While my abstract results in §2 are for metric spaces and guided by those in Prokhorov [9] for the separable case, LeCam uses a more elaborate abstract apparatus involving the second dual spaces of topological linear spaces and nonmetric topologies; the place of upper and lower integrals is taken by integrals with respect to finitely additive extensions of a measure.

With regard to the more concrete equicontinuity properties of empirical distribution functions, my approach in §4 below uses a sort of Markov property for the random empirical measure μ_n, namely given $\mu_n(E)$ for a set E, the

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