BOUNDED HOLOMORPHIC FUNCTIONS AND PROJECTIONS

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1.1. Let R be a Riemann surface and let $H^\infty(R)$ be the algebra of bounded holomorphic functions on R. I will assume that the universal covering surface of R is the open unit disc D, as it must be if $H^\infty(R)$ contains nonconstant functions, and then, because of this assumption, there are analytic maps t from D onto R such that the pair (D, t) is a regular covering surface of R [1]. Let t be one of these maps and use t to represent $H^\infty(R)$ as a subalgebra of $H^\infty(D)$ by composing the functions in $H^\infty(R)$ with t. My aim is to show, when R is conformally equivalent to the interior of a compact bordered Riemann surface, that there is a projection P of $H^\infty(D)$ onto $H^\infty(R)$ with the property

$$P(fg) = fPg$$

for all f in $H^\infty(R)$ and g in $H^\infty(D)$. By projection I mean linear and idempotent.

1.2. Let G be the group of cover transformations of (D, t). G is the group of fractional linear transformations T that take D onto D with

$$t \circ T = t,$$

and G is isomorphic to the fundamental group of the surface R. The group G acts on $H^\infty(D)$ in the standard way by composing the functions in $H^\infty(D)$ with the transformations in G, and $H^\infty(R)$ is the algebra of functions in $H^\infty(D)$ that are invariant under G. For let f be in $H^\infty(R)$ and let $g = f \circ t$ be the function in $H^\infty(D)$ that is obtained by lifting f to D. Then g is invariant under the group G,

$$Tg = g \circ T = g$$

for all T in G, and every function in $H^\infty(D)$ that is invariant under G is obtained in this way.

Each function in $H^\infty(D)$ has a radial limit at almost every point of the unit circle Γ. Let H^∞ be the algebra of functions defined almost everywhere on Γ that are radial limits of functions in $H^\infty(D)$, and let H^∞/G be the subalgebra of functions in H^∞ that are invariant under G. The radial limit map is an algebra isomorphism between $H^\infty(D)$ and H^∞ and between $H^\infty(R)$ and H^∞/G, and it is within the framework of H^∞ and H^∞/G that I will get the projection P. The arguments I will give are intrinsic in the sense that everything will take place on Γ with an occasional trip into D, and we will not need

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