ON THE NON-REGULARITY OF CERTAIN GENERALIZED LOTOTSKY TRANSFORMS

BY

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1. Introduction

The generalized Lototsky (or $[F, d_n]$) sequence to sequence transformation was defined by A. Jakimovski [3] as follows. Let $d_n = \rho_n e^{i\theta_n}$ be a fixed sequence of complex numbers ($d_n \neq -1$); then the elements P_{nk} of the $[F, d_n]$ matrix are defined by

$$P_{00} = 1, \qquad P_{0k} = 0, \quad k \neq 0$$

$$\prod_{k=1}^{n} (\theta + d_k)/(1 + d_k) = \sum_{k=0}^{\infty} P_{nk}\, \theta^k.$$

Necessary conditions that this matrix be regular are $\sum_{n=1}^{\infty} \rho_n^{-1} = +\infty$ and if $\lim_{n\to\infty} \theta_n$ exists then it must equal zero. V. F. Cowling and C. L. Miracle, [1] and [2], have shown that the additional condition

$$\sum_{n=1}^{\infty} \theta_n^2\, \rho_n^{-1} < +\infty$$

is sufficient to guarantee regularity. They also conjectured that the necessary conditions might alone be sufficient. That this is not the case is shown by an example due to A. Meir [4]. The purpose of this paper is to give some general conditions under which

$$\sum_{n=1}^{\infty} \rho_n^{-1} = +\infty \quad \text{and} \quad \lim_{n\to\infty} \theta_n = 0$$

but the $[F, d_n]$ transformation is not regular.

2. The main theorem

THEOREM 1. *Let* $\lim_{n\to\infty} \theta_n = 0$, $0 < \theta_n < \pi/2$ *for all* n. *If there exists a sequence* α_n *satisfying* $\theta_k \geq \alpha_n$ *for* $k \leq n$ *and*

$$\lim_{n\to\infty} \alpha_n^2 \sum_{k=1}^{n} \rho_k/(1 + \rho_k)^2 = +\infty$$

then the $[F, d_n]$ *transformation is not regular.*

Proof. We note that an acceptable choice for the sequence α_n is $\alpha_n = \min_{1\leq k\leq n} \theta_k$ and proceed with the proof.

Assume the $[F, d_n]$ matrix is regular. Let $\lambda_{kn} = \rho_k\, e^{i\beta_{kn}}$ where $\beta_{kn} = \theta_k - \alpha_n$ for $k \leq n$. Define the elements b_{nk} of the $[F, \lambda_{kn}]$ matrix by

$$b_{00} = 1, \qquad b_{0k} = 0, \quad k \neq 0$$

$$\prod_{j=1}^{n} (\theta + \lambda_{jn})/(1 + \lambda_{jn}) = \sum_{k=0} b_{nk}\, \theta^k.$$

The elements P_{nk} of the $[F, d_n]$ matrix are given by

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