DECOMPOSITIONS OF E^3 WITH A NULL SEQUENCE OF STARLIKE EQUIVALENT NON-DEGENERATE ELEMENTS ARE E^3

BY

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Donald V. Meyer has recently proven [4] that if an upper semi-continuous decomposition of E^3 has only a null sequence of non-degenerate elements and if each of these is a tame 3-cell then the decomposition space is E^3. We generalize this result to include any null sequence of continua provided only that each is equivalent, under a space homeomorphism, to a starlike continuum. Thus our result includes not only tame 3-cells but tame disks, triods, whiskbrooms and any combination of these.

There are still several very interesting unsolved questions in this area. For example, is the decomposition space E^3 if the non-degenerate elements

(1) form a sequence of sets, each equivalent, under a space homeomorphism, to a starlike set? This question is not answered even when each element is a tame cell.

(2) form a null sequence of strongly cellular sets [1]? i.e., for each $g \in G$ there is a cell C in E^3 with $g \in \operatorname{Int} C$, and a homotopy $f : C \times [0, 1] \to C$ such that

(a) $h(x, 0) = x$, for all $x \in C$ and $h(x, t) = x$ for all $x \in g$, $t \in [0, 1]$

(b) $h|_{BdC \times [0,1)}$ is a homeomorphism onto $C - g$

(c) $h(C \times 1) = g$.

It is known that cellular (in place of strongly cellular) in question 2 is not enough to insure that the decomposition space is E^3 [2]. The answer to question 1 is yes if each element is taken to be starlike [3].

We will use standard notation. A collection of disjoint continua G filling up E^3 is called upper semi-continuous if for each $g \in G$ and each neighborhood U of g there is a neighborhood V of g such that if $g' \in G$ and $g' \cap V \neq \emptyset$ then $g' \subset U$. The decomposition space G' is defined by letting a set $U' \subset G$ be open in G' if the set $U = \bigcup_{g \in U'} g$ is open in E^3. H denotes the collection of all non-degenerate elements of G and $H^* = \bigcup_{g \in H} g$. A continuum g is starlike with respect to $p \in g$ if every line through p intersects g in either an interval or the point p. A null sequence of sets is a sequence such that given $\varepsilon > 0$ there are only a finite number of sets in the sequence whose diameters are greater than ε.

THEOREM. *Let G be an upper semi-continuous decomposition of E^3 such that H is a null sequence of continua and each continuum $g \in H$ is equivalent* (*under a space homeomorphism*) *to a starlike continuum. Then G' is E^3.*

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