ON MOMENT SEQUENCES OF OPERATORS

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1. Introduction

Let X, Y be Banach spaces over the complex field and denote by $B \equiv B(X, Y)$ the space of continuous linear operators on X into Y. Recently Tucker [6] has introduced a weak extension Y^+ of the Banach space Y and has proved that $B^+ \subseteq B(X, Y^+)$. The weak extension Y^+ is by construction a subspace of Y^{**}, consequently if $\overline{B^+}$ denotes the closure of B^+ in $B^{**}(X, Y)$ topologized in the natural way we obtain $\overline{B^+} \subseteq B(X, Y^{**})$.

DEFINITION 1. Given a sequence $\{\psi_n(t)\}(n \geq 0) \subseteq C[0, 1]$, the sequence $\{A_n\} \subseteq B(X, Y)$ is called a weak moment sequence with respect to $\{\psi_n(t)\}$ if there exists a vector-valued measure μ, defined on the σ-field of Borel sets in $[0, 1]$ into $\overline{B^+}$ such that

(i) $\mu(\cdot)b^*$ is in rca $[0, 1]$ for each $b^* \in B^*(X, Y)$;
(ii) the mapping $b^* \to \mu(\cdot)b^*$ is continuous with the $B(X, Y)$ and $C[0, 1]$ topologies of $B^*(X, Y)$ and rca $[0, 1]$ respectively;
(iii) $b^*A_n = \int_0^1 \psi_n(t)\mu(dt)b^* \quad n = 0, 1, 2, \cdots, b^* \in B^*(X, Y)$;
(iv) $\| \mu \|[0, 1] = \sup \| \sum \alpha_i \mu(E_i) \| < \infty$,

where the supremum is taken over all finite collections of disjoint Borel sets in $[0, 1]$ and all finite sets of scalars α_i with $| \alpha_i | \leq 1$.

DEFINITION 2. Given a sequence $\{\psi_n(t)\} \subseteq C[0, 1]$, the sequence $\{A_n\} \subseteq B(X, Y)$ is called a strong moment sequence with respect to $\{\psi_n(t)\}$ if there exists a vector-valued measure μ, defined on the σ-field of Borel sets in $[0, 1]$ into $B(X, Y)$ such that

(i) $b^*\mu(\cdot)$ is in rca $[0, 1]$, $b^* \in B^*(X, Y)$;
(ii) $A_n = \int_0^1 \psi_n(t)\mu(dt) \quad n = 0, 1, 2, \cdots$;
(iii) $\| \mu \|[0, 1] < \infty$.

(For definitions and details see [2].)

It is our purpose to obtain necessary and sufficient conditions on a sequence $\{A_n\}$ $(n \geq 0)$ of operators in $B(X, Y)$ in order that it will be a weak or a strong moment sequence with respect to $\{\psi_n(t)\}$ $(n \geq 0)$ in various cases of sequences $\{\psi_n(t)\}$. We shall be interested, especially, in the case where $\psi_n(t) = t^{\lambda_n}$, $n \geq 0$, where the sequence $\{\lambda_n\}$ $(n \geq 0)$ satisfies

$$(1.1) \quad 0 \leq \lambda_0 < \lambda_1 < \cdots < \lambda_n < \cdots \uparrow \infty, \qquad \sum_{i=1}^{\infty} 1/\lambda_i = \infty.$$

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