A CHARACTERIZATION OF SOME MULTIPLY TRANSITIVE PERMUTATION GROUPS, I

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The objective of this paper is to give a proof of the following result:

Theorem A. *Let G be a finite simple group which contains an involution t such that the following conditions are satisfied:*

(I) *The centralizer $\mathbf{C}_G(t)$ of t in G is a splitting extension of an elementary abelian normal 2-subgroup of order at most 16 by S_4, the symmetric group of degree four;*

(II) *the centre of a Sylow 2-subgroup of $\mathbf{C}_G(t)$ is cyclic.*

Then G is isomorphic to one of the following groups A_8, A_9, A_{10} or M_{22}. Here A_n denotes the alternating group of degree n, and M_{22} is the Mathieu simple group on 22 letters.

This result is a consequence of the following

Theorem B. *Let π_0 be an involution contained in the centre of a Sylow 2-subgroup of A_{10}. Denote by H_0 the centralizer of π_0 in A_{10}.*

Let G be a finite group with the following two properties:

(a) *G has no subgroups of index 2, and*

(b) *G possesses an involution π such that the centralizer $\mathbf{C}_G(\pi)$ of π in G is isomorphic to H_0.*

Then G is isomorphic to A_{10}.

Remark. Let G be a group satisfying the assumptions of Theorem A. Then $\mathbf{C}_G(t)$ contains an elementary abelian normal 2-subgroup M of order at most 16 such that $\mathbf{C}_G(t)$ is a splitting extension of M by S_4. Hence $|M|$ is equal to 8 or 16. It is straightforward to check, that, if $|M| = 8$, then $\mathbf{C}_G(t)$ is uniquely determined. Application of the result in [8] yields that G is isomorphic to A_8 or A_9 if $|M| = 8$. However, if $|M| = 16$, there are precisely two possibilities for $\mathbf{C}_G(t)$ as has been observed in [10]. One of these possibilities is that $\mathbf{C}_G(t)$ is isomorphic to the centralizer H_1 of an involution of M_{22}, the other possibility is that $\mathbf{C}_G(t)$ is isomorphic to the centralizer of an involution of A_{10}. The theorem in [10] states that if $\mathbf{C}_G(t)$ is isomorphic to H_1 then G is isomorphic to M_{22}. Hence, in order to prove Theorem A, it suffices to prove Theorem B.

1. Some properties of H_0

The group H_0 is isomorphic to a group H generated by the elements π, μ,

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