SOME SUBGROUPS OF $SL_n(\mathbf{F}_2)$

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1. Introduction

In this paper we determine those irreducible subgroups of $SL_n(\mathbf{F}_2)$ which are generated by transvections.

THEOREM. *Let V be a vector space of dimension $n \geqq 2$ over $\mathbf{F}_2$ and let G be an irreducible subgroup of $SL(V)$ which is generated by transvections. If $G \neq SL(V)$ then $n \geqq 4$ and G is one of the following subgroups of $Sp(V)$: $Sp(V)$, $O_{-1}(V)$, $O_1(V)$ (except at $n = 4$), the symmetric group of degree $n + 2$, or the symmetric group of degree $n + 1$.*

This result has some relevance to the question left open in [3].

Some of the notation and terminology of [3] will be used and we review it briefly there. (Since we work over a finite prime field our assumption that G is generated by transvections is equivalent to the assumption that G is generated by subgroups of root type.) If G contains the transvection τ with $P = \operatorname{Im}(\tau - 1)$ and $H = \operatorname{Ker}(\tau - 1)$ we say P is a *center* (for G), H is an *axis* (for G). Also we say P is a center for H and H is an axis for P. The set of centers for G is C and the set of axes for G is A. For $P \in C$, $a(P)$ is the intersection of the axes of P and for $H \in A$, $c(H)$ is the sum of the centers for H.

2. Preliminary lemmas

Our determination will be made by induction on n; in this section we collect some information needed for the induction. G is a group satisfying the hypotheses of the theorem.

LEMMA 2.1. *G is transitive on C and A.*

Proof. Choose P such that $\dim a(P)$ is maximal. Then Lemma 2 of [3] tells us that G has an orbit of centers containing P and all centers off $a(P)$. Since G is irreducible there cannot be a second orbit. Likewise for A.

LEMMA 2.2. *If $P \in C$ and $a(P)$ is not a hyperplane then $G = SL(V)$.*

Proof. Choose $P \in C$ and suppose S is another center on $a(P)$. By Lemma 4 of [3] we have a center Q off $a(P)$ and $a(S)$. Let K be a hyperplane over $Q + a(P)$. Since $K \supseteq a(P)$, K is an axis for P. Then using Lemma 2 of [3] we see K is an axis for Q and then K is an axis for S. Thus all points on $P + S$ are centers. Since G is irreducible, C spans V and consequently every

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