ON THE SIMPLE GROUP OF D. G. HIGMAN AND C. C. SIMS

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1. Introduction

Higman and Sims [3] describe their group as a permutation group of rank 3 and degree 100, in which the stabiliser of a point is a Mathieu group M_{22}, and has orbits of lengths 1, 22 and 77. Here we shall exhibit what is presumably the same group as a doubly transitive permutation group of degree 176, in which the stabiliser of a point is an extension by a field automorphism of the projective unitary group $PSU(3, 5^2)$. (We shall denote this extension by $P\Sigma U(3, 5^2)$.)

We shall construct a "geometry" of which the group is the automorphism group. We shall call the objects of the geometry points, conics and quadrics, since these seem to be the simplest objects of ordinary geometry which could possibly realize the required configuration; though I do not know that it can be realized in any projective space. The geometry has the following properties.

(i) There are 176 points, 1100 conics and 176 quadrics.

(ii) Each quadric contains 50 points, and each point is on 50 quadrics.

(iii) Each conic contains 8 points and lies on 8 quadrics.

(iv) Through any two points there pass just two conics; any quadric through both points contains at least one of the conics; and just two quadrics contain both conics.

(v) On any two quadrics there lie just two conics; any point on both quadrics lies on at least one of the conics; and there are just two points lying on both conics.

(vi) A conic S determines a one to one correspondence between the points q on it and the quadrics Q through it, such that, if q corresponds to Q,

(α) The conics S' meeting S in two points, one of which is q, lie on Q;

(β) The conics S' lying on two quadrics through S, one of which is Q, contain q.

The automorphism group of the geometry is transitive on conics, on incident point-quadric pairs, and on non-incident point-quadric pairs, and doubly transitive on points and on quadrics. Thus the stabiliser of a point permutes the quadrics in orbits of lengths 50 and 126. On the orbit of length 50, it is a rank 3 group with suborbits of lengths 1, 7 and 42, and so by a result of D. G. Higman [2] is either $PSU(3, 5^2)$, of order 126,000, or $P\Sigma U(3, 5^2)$ of order 252,000. However, the stabiliser of a point-pair is fairly easily seen to have order 2,880, being isomorphic to $Z_2 \times \mathrm{Aut}\,(A_6)$, which implies that the stabiliser of a point has order 252,000, and that the whole group has order