SPECTRAL REPRESENTATIONS FOR A GENERAL CLASS OF OPERATORS ON A LOCALLY CONVEX SPACE

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1. Introduction

The material in this paper is a generalization of the work of Maeda [15] which in turn generalized the work of Foias [8], C. Ionescu Tulcea [10], and Dunford [6].

Throughout the paper the symbol N will denote the set $\{0, 1, 2, \cdots\}$, Z the set of integers, R the set of real numbers, R^2 or C the Euclidean (complex) plane, and T^1 the unit circle in R^2 considered as a one-dimensional manifold. For any subset S of R^2, $\mathfrak{K}(S)$ will be the set of all compact subsets of R^2 contained in S; $\mathfrak{K}$ will denote $\mathfrak{K}(R^2)$.

For a non-empty open subset Q of C the algebra $\mathcal{H}(Q)$ of complex-valued functions holomorphic on Q will be endowed with the topology of uniform convergence on compact subsets of Q. A *holomorphic function over* $K \in \mathfrak{K}$ is a function holomorphic over some open neighborhood of K. Two holomorphic functions f and g over K are equivalent if $f \mid Q = g \mid Q$ for some neighborhood Q of K. The set $\mathcal{H}(K)$ of equivalence classes of functions holomorphic over K is considered as an algebra in the natural way. When endowed with the "van Hove topology" (the inductive limit topology induced by the natural mappings of $\mathcal{H}(Q)$ into $\mathcal{H}(K)$), $\mathcal{H}(K)$ is a topological algebra with unit 1. The symbol $\lambda(\lambda \in C)$ will denote the element $\lambda 1$ of $\mathcal{H}(K)$; the symbol z, the identity function of R^2 onto itself considered as an element of $\mathcal{H}(K)$. Similarly, for $\lambda \in \mathbf{C}K(= C\backslash K)$, the function ψ_λ defined by the equation $\psi_\lambda(z) = 1/(\lambda - z)$ is the inverse of $(\lambda - z)$ in $\mathcal{H}(K)$. The basic properties of $\mathcal{H}(Q)$ and $\mathcal{H}(K)$ are discussed in [21].

All vector spaces will be over the complex field C. If E and F are vector spaces, let $\mathcal{L}^*(E, F)$ be the set of linear mappings of E into F; if E and F are topological vector spaces, let $\mathcal{L}(E, F)$ be the set of continuous linear mappings of E into F. Whenever E is a separated locally convex space, $\mathcal{L}(E)(=\mathcal{L}(E, E))$ will be assumed endowed with the topology of uniform convergence on sets of a family of bounded sets in E. $\mathcal{L}(E)$ is an algebra whose identity element will be denoted by I. Let $E' = \mathcal{L}(E, C)$, the (topological) dual of E.

If $T \in \mathcal{L}^*(D_T, E)$ where D_T is a subspace of E, one says that T is a transformation or mapping defined *in* E. A transformation T in E is said to have the *single-valued extension property* [6] if, for any open subset Q of C and any

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