# A LIMITATION THEOREM FOR CESÀRO SUMMABLE SERIES

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## 1. Introduction

We consider the Cesàro summability, for integral orders of the series $\sum_{\nu=0}^{\alpha} a_\nu\, d_\nu$. In this paper we establish a limitation theorem for this series. Results of this character, but not overlapping with those in this paper, were given by Hardy and Littlewood [7] and by Andersen [1]. Andersen's result was extended by Bosanquet and Chow [5], and further extended by Bosanquet [4].

*Notation.* We write

$$A_n^0 = A_n = a_0 + a_1 + \cdots + a_n, \quad A_n^k = A_0^{k-1} + A_1^{k-1} + \cdots + A_n^{k-1}$$

and we get the identities [6]

$$A_n^k = \sum_{\nu=0}^{n} \binom{n-\nu+k-1}{k-1} A_\nu, \quad A_n^k = \sum_{\nu=0}^{n} \binom{n-\nu+k}{k} a_\nu, \quad E_n^k = A_n^k$$

when $a_0 = 1$, $a_n = 0$, for $n > 0$ i.e. when $A_n = 1$, for all $n$. So

$$E_n^k = \binom{n+k}{k} \sim \frac{n^k}{k!}.$$

$\sum a_n$ is said to be summable $(C, k)$ to $A$ if $A_n^k/E_n^k \to A$ as $n \to \infty$, or equivalently if $k!\, A_n^k/n^k \to A$.

We write $\Delta d_n = d_n - d_{n-1}$, following L. S. Bosanquet [3]. We will use the following identity (see L. S. Bosanquet [3]):

$$\Delta^k (U_n V_n) = \sum_{\nu=0}^{k} \binom{k}{\nu} \Delta^\nu U_n\, \Delta^{k-\nu} V_{n-\nu}. \tag{1.1}$$

## 2. Statement of the theorem and two lemmas.

THEOREM 1. *Suppose that $d_n > 0$, for $n \geq 0$, and*

(i) $d_{n+1} = o(1)$ *as* $n \to \infty$,

(ii) $$\frac{d_{n+1}}{n^k} \sum_{\nu=0}^{n} \nu^k \binom{n-\nu+k}{k} \frac{1}{d_{\nu+k+1}} = O(1),$$

(iii) $|\Delta^j(1/d\nu + k + 1)| \leq K |\Delta^{j-1}(1/d_\nu + k + 1)|$,

$j = 1, 2, \cdots, k+1$; $k \geq 0$, *$k$ an integer*; $\Delta$ *operating on* $\nu$.

*Then* $A_n^k = o(n^k/d_{n+1})$ whenever $\sum_{\nu=0}^{\infty} a_\nu\, d_\nu$ is summable $(C, k)$.

We require the following lemmas.

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