REPRESENTATION OF A COMPLEMENTED ALGEBRA ON A LOCALLY COMPACT SPACE

BY

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1. It was shown in [8] that for each simple complemented algebra A there exists a measure space (S, μ) and a real-valued positive function $k(s)$ on S such that A is isomorphic to the set of all measurable functions $f(s, t)$ on $S \times S$ for which the expression $\int\int |f(t, s)|^2 k(s)\, d\mu(t)\, d\mu(s)$ is finite. In this note we intend to show that there is a certain very natural topology τ on S with respect to which the function $k(s)$ is continuous almost everywhere and the measure μ is a Radon measure.

2. Let A be a Banach algebra whose underlying Banach space is a Hilbert space. Then A is called a complemented algebra [6] if the orthogonal complement of every right (left) ideal of A is again a right (left) ideal. To exclude the trivial case, when the product of any two members of the algebra is zero, we assume all algebras in the paper to be semi-simple.

We use the term *Radon measure* to refer to a measure on a locally compact Hausdorff space which corresponds to an integral on the set of all complex-valued continuous functions with a compact support (in the way it does, for example, in §6 of Naimark's book [5]). (An explicit definition of a Radon measure can be found in [4, page 9].) We assume that the reader is familiar with §6 of [5] and we are going to use the terminology of this section of Naimark's book.

Below is an example of a simple complemented algebra (compare with the example in [8]).

Example. Let (S, τ) be a locally compact Hausdorff space and let μ be a Radon measure on S. Let $k(s)$ be a measurable real-valued function on S bounded below by a positive number and finite except on a locally zero set [5, page 131]. (In particular $k(s)$ may be continuous at each point in S at which it is finite.) Let A be the set of all complex-valued measurable functions $x(t, s)$ on $S \times S$ such that $\int\int |x(t, s)|^2 k(s)\, d\mu(t)\, d\mu(s) < \infty$. Then A is a complemented algebra with respect to the multiplication

$$(xy)(t, s) = \int x(t, r)y(r, s)\, d\mu(r)$$

and the scalar product $(x, y) = \int \int x(t, s)\bar{y}(t, s)k(s)\, d\mu(s)\, d\mu(t)$.

If $k(s)$ is essentially bounded then A is a two-sided H^*-algebra.

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