TOPOLOGICAL SEMIGROUPS AND FIXED POINTS

BY

THEODORE MITCHELL[1]

1. Introduction

In this paper, we consider four fixed point properties that a topological semigroup S might conceivably possess.

($F1$) Whenever S acts on a compact Hausdorff space Y, where the map $S \times Y \to Y$ is jointly continuous, then Y contains a common fixed point of S.

($F2$) Whenever S acts affinely on a convex compact subset Y of a locally convex linear topological space, where the map $S \times Y \to Y$ is jointly continuous, then Y contains a common fixed point of S.

($F3$) Whenever S acts on a compact Hausdorff space Y, where the map $S \times Y \to Y$ is separately continuous, then Y contains a common fixed point of S.

($F4$) Whenever S acts affinely on a convex compact subset Y of a locally convex linear topological space, where the map $S \times Y \to Y$ is separately continuous, then Y contains a common fixed point of S.

For each of these four properties, we investigate the question as to whether there exists some subspace of $C(S)$ whose left amenability (or whose extreme left amenability) is equivalent to the specified (Fi). It is shown that for each of the (Fi), there does indeed exist such an associated subspace of $C(S)$; in fact, a total of three spaces will suffice to characterize the four properties in this manner. Let $f \in C(S)$, and define $\theta_f : S \to C(S)$ by $\theta_f s = l_s f$ for $s \in S$. Then we will say $f \in LUC(S)$ ($f \in WLUC(S)$) $\{f \in LMC(S)\}$ if the map θ_f is continuous when $C(S)$ is given the supremum norm topology (w-topology) {weak topology induced by the multiplicative means on $C(S)$}. The following are shown in Sections 3 and 4

THEOREM 1. *S satisfies ($F1$) iff $LUC(S)$ has a multiplicative left invariant mean.*

THEOREM 2. *S satisfies ($F2$) iff $LUC(S)$ has a left invariant mean.*

THEOREM 3. *S satisfies ($F3$) iff $LMC(S)$ has a multiplicative left invariant mean.*

THEOREM 4. *S satisfies ($F4$) iff $WLUC(S)$ has a left invariant mean.*

The concept of characterizing fixed point properties of topological semi-

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