NOTE ON QUASIFIBRATIONS AND FIBRE BUNDLES

BY

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1. Introduction

In a previous paper of the authors' [4], the problem of classifying rank 2 H-spaces up to homotopy type was considered. The methods used in that paper led to the study of CW-complexes of the form $X = S^q \cup_\alpha e^n \cup_\beta e^{n+q}$ which are quasifibrations over S^n in the sense of Dold-Thom [1], i.e. there exists a map of pairs

$$p : (X, S^q) \to (S^n, \text{point})$$

inducing isomorphisms $p_* : \pi_i(X, S^q) \to \pi_i(S^n)$ for all i.

In [4], our major interest was in the cases $q = 3$ and $n = 5$ or 7; as a by-product of our main results, the following theorem was obtained.

THEOREM 1.1. *If $X = S^q \cup_\alpha e^n \cup_\beta e^{n+q}$ quasifibres over S^n with $q = 3$ and $n = 5$ or 7, then X has the homotopy type of an orthogonal S^q-bundle over S^n.*

This theorem does not generalize to other values of q and n; in fact, a specific counterexample has been given by Sutherland [11] with $q = 3$, $n = 8$. Our purpose here is to provide a whole family of examples of S^q-quasifibrations over S^n which are not homotopy equivalent to orthogonal S^q-bundles over S^n. In all of our examples, we will have $q = 2$ and the first attaching map $\alpha = 0$; the latter implies the existence of a "cross-section" for the quasifibration.

Sutherland constructs his example so that the total space has the homotopy type of a closed, smooth manifold. He then shows that his example does not even have the homotopy type of a differentiable S^3-bundle over S^8, i.e. a fibre bundle over S^8 with fiber S^3 and structural group Diff (S^3), the group of diffeomorphisms of S^3. In fact, Sutherland observes that results from Cerf's thesis can be used to show that any differentiable S^q-bundle over S^n must be fibre homotopy equivalent to an orthogonal S^q-bundle over S^n.

Many of the examples we construct also turn out to be homotopy equivalent to closed, smooth manifolds so that the above remark applies to these examples. However, we can even go one step further and assert that none of our examples has the homotopy type of an S^2-fibre bundle over S^n, with structural group the full group of homeomorphisms of the fibre. This is due to the classical fact that the full group of homeomorphisms of S^2 has the homotopy type of the 3-dimensional orthogonal group.

One might enquire whether the manifolds we construct here have a reasonable "geometric" description. We do not attempt to give such a description for all our examples, but we do succeed in doing this for our lowest dimensional

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