EXTENSION OF SPECTRAL MEASURES

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The main object of the present paper is the study of the resolutions of the identity of the sum and the product of two commuting scalar type operators on an arbitrary Banach space. This is achieved by studying for an arbitrary Banach space the extension of a spectral measure defined on a field R of subsets of a set $\mathfrak{S}$ to the σ-field generated by R. A very special aspect of this study is the content of a recent paper by Kluvánek and Kovářiková [11]. These authors have restricted their attention to the case where the spectral measure to be extended is the product of two commuting spectral measures and the Banach space is weakly complete. Again their extension assumes a topological set up on $\mathfrak{S}$. Naturally the results of the present paper subsume those of [11].

The procedure of closely following the numerical analogue is used in the present paper; the referee has pointed out that the extension can also be obtained by reducing matters to the case of Hilbert space. In the latter method the extension rests on known results (Berberian [4]) which are however established for Hilbert spaces in [4] by a method altogether different from that adopted in this paper. The aesthetic satisfaction in sticking to the Banach space alone, as in this paper has justification on two counts: (i) the auxiliary notion of spectral outer measure introduced here seems to be interesting and worthwhile in itself, (ii) the fact that results follow from their numerical analogues is brought out vividly.

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1. Preliminaries

X will denote an arbitrary (complex) Banach space. A B. A. B of projections on X will be called complete (σ-complete) if B satisfies the condition of Definition 2.1 of Bade [2].

In this section, we recall some definitions and results from [14].

DEFINITION 1. By a $W^*(\|\cdot\|)$-algebra W on a Banach space X we mean a pair, consisting of an abelian subalgebra W of $B(X)$, generated by a σ-complete B.A. of projections on X in the weak operator topology, and some equivalent norm $\|\cdot\|$ on X such that every element S in W has a representation of the form $S = R + iJ$ where R and J satisfy the following conditions:

(i) $RJ = JR$ with R and J in W;

(ii) R^mJ^n $(m, n = 0, 1, 2, \cdots)$ are hermitian in the norm $\|\cdot\|$ (hermitian in the sense of Lumer [12]).

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