ON MIXING AND PARTIAL MIXING

BY

N. A. FRIEDMAN AND D. S. ORNSTEIN[1]

1. Introduction

Let $(X, \mathfrak{A}, m)$ denote the unit interval with Lebesgue measure, and let τ be an invertible ergodic measure preserving transformation on X. τ is mixing if

$$\lim_n m(A \cap \tau^n B) = m(A)m(B), \qquad A \text{ and } B \text{ in } \mathfrak{A}. \tag{1.1}$$

Given $\alpha > 0$, τ is partially mixing for α if

$$\lim_n \inf m(A \cap \tau^n B) \geqq \alpha m(A)m(B), \qquad A \text{ and } B \text{ in } \mathfrak{A}. \tag{1.2}$$

In [3], a transformation τ is constructed such that τ is partially mixing for $\alpha = \frac{1}{8}$ but τ is not mixing. It is easily verified that τ is mixing if and only if τ is partially mixing for $\alpha = 1$.

The results in this paper are in two parts. The first result is concerned with mixing transformations. Let τ be mixing, $f \in L_1$, and let (k_n) be an increasing sequence of positive integers. Define f_n and $E(f)$ as

$$f_n(x) = (1/n) \textstyle\sum_{i=1}^{n} f(\tau^{k_i}(x)), \qquad E(f) = \int f\, dm.$$

In [1], Blum and Hanson proved that f_n converges to $E(f)$ in the mean. In §4, we construct an example such that for a. e. x, $f_n(x)$ does not converge pointwise.

The second result concerns partial mixing transformations. In §5, it is shown that given $\alpha \in (0, 1)$, there is an explicit construction of a transformation τ such that τ is partially mixing for α but τ is not partially mixing for any $\alpha + \varepsilon$, $\varepsilon > 0$.

Both of the above results are based on a construction given in §3. Some preliminary results are given in §2. We shall utilize notation and terminology n [2].

2. Preliminaries

In [2], [3], the S operator was defined for a tower with columns of equal width. The definition will now be extended to the case where the columns generally have unequal widths. Let

$$T = \{C_j : 1 \rightarrow j \rightarrow q\} \quad \text{where } C_j = (I_{j,k} : 1 \rightarrow k \rightarrow h_j).$$

The intervals in C_j have the same width $w_j(T)$. The top of T is

$$A(T) = \textstyle\bigcup_{j=1}^{q} I_{j,h_j},$$

Received June 1, 1969.

1 Research supported by a National Science Foundation grant.