ON DOUBLY TRANSITIVE PERMUTATION GROUPS OF DEGREE $n \equiv 2 \bmod 4$

BY

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Let G be a doubly transitive permutation group on a set Ω of degree $n \equiv 2 \bmod 4$, $n > 2$. Let α and β be distinguished and distinct points in Ω, and let $H = G_\alpha$, $D = G_{\alpha\beta}$, and T be a Sylow 2-subgroup of D. We prove the following two results:

THEOREM 1. *G contains a unique minimal normal subgroup $M(G)$. $M(G)$ is simple and doubly transitive on Ω, with $G \leq \operatorname{Aut} M(G)$.*

THEOREM 2. (*i*) *if u is an involution in T such that the fixed point set of every element of $T^{\#}$ is contained in that of u, then $M(G)^{\Omega}$ is A_6, $L_2(q)$, or $U_3(q)$ in their natural doubly transitive representations.*

(*ii*) *if T is abelian then $M[G]^{\Omega}$ is an in* (i).

Theorem 1 reduces the problem of determining all groups mentioned in the title, or any subclass thereof, to the problem of finding all such simple groups. A natural question that arises is which doubly transitive groups satisfy theorem 1. The only groups known to the author that do *not* are those with regular normal subgroups, and the Ree group, $R(3)$.

To date most characterizations of doubly transitive groups seem to be in terms of the structure of the stabilizer of two points, and particular its involutions. Theorem 2 is a result of this kind. Lemmas 3 and 4 are useful in general problems of the sort discussed in this paragraph.

1. Throughout this paper G, H, T, etc. are as above. For $X \subseteq G$ we write $F(X)$ for the set of ω in Ω with $\omega x = \omega$ for every x in X. More generally our permutation group theoretic notation is as in [5].

A_n is the alternating group on n letters. $L_2(q)$, $U_3(q)$, are the simple normal subgroups of the two, three, dimensional projective special linear, unitary, group over $GF(q)$, respectively.

We require the following two results, which we quote without proof.

LEMMA 1 (Witt, [6]). *Let A be a t-transitive permutation group on $\Omega = \{1, 2, \cdots, n\}$ and let B be the subgroup of A fixing each i, $1 \leq i \leq t$. Let $U \subseteq B$. Then $(N_G U)^{F(U)}$ is t-transitive if and only if for a in A, $U^a \subseteq B$ implies there exists b in B with $U^a = U^b$.*

LEMMA 2 (Suzuki, [4]). *Let U be a 2-group and u an involution in U such that $C_U(u)$ is the four group. Then U is dihedral or semidihedral*

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