A NOTE ON THE STONG-HATTORI THEOREM

BY

LARRY SMITH

Summary. The theorem of Stong and Hattori referred to in the title asserts that the natural mapping

$$sh : \Omega_*^U(X) \to K_*(X \wedge MU)$$

is a split monomorphism whenever X is a finite complex with torsion free integral homology. In the present note we will show that the map sh remains monic (although need no longer be split) for those complexes with $\Omega_*^U(X)$ of projective dimension at most one as an Ω_*^U-module. Two proofs will be presented—one for K-theory and one for k-theory. We show by example that the result is best possible.

Let us begin by fixing our notations and conventions. We assume that we are working in a suitable category of spectra where the $\wedge$ product is defined, such as that constructed by Boardman [9]. We denote by $\Omega_*^U(\quad)$ the complex bordism homology theory which is represented by the Thom spectrum $\mathbf{MU}$. We write $K_*(\quad)$ for the homology theory dual to the usual K-cohomology theory, and $k_*(\quad)$ for the homology theory represented by the connective $\mathbf{bu}$ spectrum. The representing spectrum for $K_*(\quad)$ is denoted by $\mathbf{BU}$. There is the natural commutative diagram of spectra

$$\begin{tikzcd} & \mathbf{bu} \wedge \mathbf{MU} \arrow[dd, "\lambda \wedge 1"] \\ \mathbf{MU} \arrow[ur, "sh"] \arrow[dr, "SH"'] & \\ & \mathbf{BU} \wedge \mathbf{MU} \end{tikzcd}$$

which for any finite complex X yields a commutative diagram

$$\begin{tikzcd} & k_*(X \wedge MU) \arrow[dd, "\lambda"] \\ \Omega_*^U(X) \arrow[ur, "sh"] \arrow[dr, "SH"'] & \\ & K_*(X \wedge MU). \end{tikzcd}$$

Note that $sh = (1_{\mathbf{bu}} \wedge 1)_*$ and similarly for SH.

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