# PARABOLIC POTENTIALS WITH SUPPORT ON A HALF-SPACE

BY
RICHARD J. BAGBY

## 1. Introduction

We study the class of parabolic potentials $\mathcal{L}_\alpha^p$ introduced by Jones [4]. These spaces arise in the study of the heat equation; they are analogous to Sobolev spaces of fractional order.

We direct our attention to the problem of deciding whether the restriction of a function in $\mathcal{L}_\alpha^p$ to a half-space necessarily agrees with a function in $\mathcal{L}_\alpha^p$ supported on that half-space. In the case of Sobolev spaces the result is well known; one method of answering this question appears in Strichartz [7, §3]. Essentially the same approach is used here, but the presence of the time variable raises a number of complications.

For $1 < p < \infty$, it is possible to describe $\mathcal{L}_\alpha^p$ in terms of Sobolev spaces on $R$. This is done, for example, in [2]. Such a characterization could also be used here to give a somewhat shorter proof of the main theorem. However, the techniques used here produce additional insight.

## 2. Definitions and basic properties

DEFINITION. A function $f$ is in $\mathcal{L}_\alpha^p(R^{n+1})$ if $\hat{f} = (1 + |x|^2 + it)^{-\alpha/2}\hat{\phi}$ for some $\phi \in L^p(R^{n+1})$. Here $x \in R^n$, $t \in R$, and $\wedge$ denotes the Fourier transform in $R^{n+1}$. The norm of $f$ is $\|f\|_{p,\alpha} = \|\phi\|_p$.

DEFINITION.

$$\begin{aligned} H_\alpha(x, t) &= t^{(\alpha-n)/2-1} \exp\{-x^2/4t\}, & t > 0 \\ &= 0, & t \le 0 \end{aligned}$$

Sampson [5] proves that if $f \in \mathcal{L}_\alpha^p$, $0 < \alpha < n + 2$, then $f = H_\alpha * g$ for some $g \in L^p$ with $\|g\|_p \le c_{p,\alpha}\|f\|_{p,\alpha}$.

The following functional is useful in examining these spaces:

$$S_\alpha f(x, t) =$$

$$\left\{\int_0^\infty \left[\int_{|y|<1}\int_0^1 |f(x - ry, t - r^2s) - f(x, y)|\, ds\, dy\right]^2 r^{-1-2\alpha}\, dr\right\}^{1/2}.$$

Theorem 2.2 of [1] states that for $0 < \alpha < 1$ and $1 < p < \infty$, $f \in \mathcal{L}_\alpha^p$ if and only if both $f \in L^p$ and $S_\alpha f \in L^p$, and that $\|f\|_{p,\alpha} \approx \|f\|_p + \|S_\alpha f\|_p$. Since

$$S_\alpha(fg) \le \|f\|_\infty S_\alpha g + |g| S_\alpha f,$$

this characterization of $\mathcal{L}_\alpha^p$ is especially useful when products of functions are involved.

Received October 3, 1972.