ON THE ALGEBRAIC STRUCTURE OF THE K-THEORY OF $\frac{G_2}{SU(3)}$ AND $\frac{F_4}{Spin(9)}$

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This paper is an extension of the results of [8] to the exceptional Lie groups G_2 *and* F_4. In [8] we discussed the following situation. Suppose G is a compact connected Lie group and H is a subgroup of maximal rank. We let $R(G)$ and $R(H)$ denote the *complex representation rings* of G and H respectively [1], [6]. We can think of $R(G)$ as a subring of $R(H)$ [6] making $R(H)$ an $R(G)$ module.

An extension of the Weyl character formula yields a *duality homomorphism*,

$$F : R(H) \to \operatorname{Hom}_{R(G)}(R(H), R(G)),$$

and this was shown in [8] to be an isomorphism for a large number of cases involving the classical groups.

Among the corollaries of this theorem is a new proof of the conjecture by Atiyah-Hirzebruch [2] that $\alpha : R(H) \to K(G/H)$ is onto. We are also able to derive an explicit free basis for generating $R(H)$ over $R(G)$. This in turn yields an explicit basis for the free abelian group $K(G/H)$ [8, §9].

For those more familiar with equivariant K-theory we know that $R(H) \cong K_G(G/H)$, $R(G) \cong K_G(\text{point})$ [7]. The theorem can then be thought of as a Poincare duality result for this cohomology theory.

1. Let G be a compact connected Lie group and H a subgroup of maximal rank. That is H contains a maximal torus, T, of the group G. We can form the complex representation ring of G, denoted $R(G)$ [1], [6]. As a group $R(G)$ is the free abelian group on the set of isomorphism classes of irreducible complex representations of G, with the ring structure induced by the tensor product of representations. Restriction of representations makes $R(G)$ in a natural way a subring of $R(H)$, and $R(H)$ a subring of $R(T)$ [6]. We also think of each ring as a module over its subrings.

If $T \cong S^1 \times \cdots \times S^1$ (n times) then $R(T) \cong Z[x_1, \cdots, x_n, x_1^{-1}, \cdots, x_n^{-1}]$ the polynomial ring over the integers in n indeterminates and their inverses [1]. To each group G, H is associated a group of automorphisms of T (hence of $R(T)$) called the Weyl group, denoted $W(G)$, $W(H)$ respectively. A major theorem in the representation theory of Lie groups asserts that $R(G)$ is the fixed subring of $R(T)$ under the action of $W(G)$ [1].

It is well known [1] that each element of the Weyl group can be given a sign, $(-1)^\sigma = \pm 1$, for $\sigma \in W(G)$. An alternating operator, A, can then be defined for all $x \in R(T)$ by $A(x) = \sum_{\sigma \in W(G)} (-1)^\sigma \sigma(x)$ [1], [8].

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