ON THE DIMENSION OF VARIETIES OF SPECIAL DIVISORS, II[1]

BY

R. F. LAX

1. Introduction

In [1], we defined the analytic spaces $\mathscr{G}_n^r$, the universal analytic spaces of special divisors. We derived the equations which define the tangent space at a point of $\mathscr{G}_n^r - \mathscr{G}_n^{r+1}$. Let X be a compact Teichmuller surface of genus g and suppose s_0 is the module point of X on T_g, the Teichmuller space. Let D be a divisor on X of degree n and dimension r and let i denote the index of specialty of D. With the notation of [1], the tangent space to $\mathscr{G}_n^r$ at (s_0, D) is defined by ir equations $E_{j,k}$, where $j = 1, \ldots, r$ and $k = 1, \ldots, i$, in the $3g - 3 + n$ unknowns $s_1, \ldots, s_n, b_1, \ldots, b_{3g-3}$. The coefficient of b_m in $E_{j,k}$ is given by evaluating $\alpha_{j,k}$, a certain quadratic differential depending on D, at a point Q_m on X, which is chosen to satisfy certain requirements.

PROPOSITION 1. *Suppose (s_0, D) is in $\mathscr{G}_n^r - \mathscr{G}_n^{r+1}$ and suppose that $ir \leq 3g - 3$. Put $\tau = (r + 1)(n - r) - rg$. Then if all the $\alpha_{j,k}$ are linearly independent, the dimension of the tangent space to $\mathscr{G}_n^r$ at (s_0, D) is $3g - 3 + \tau + r$ and (s_0, D) is a smooth point of $\mathscr{G}_n^r$.*

Proof. [1].

In [1], we showed that $\mathscr{G}_n^1 - \mathscr{G}_n^2$, if nonempty, is smooth of pure dimension $3g - 3 + \tau + 1$. In this paper, by explicit computations, we show that $\mathscr{G}_n^2$ (resp. $\mathscr{G}_n^3$) has a component of dimension $3g - 3 + \tau + 2$ (resp. $3g - 3 + \tau + 3$) if τ is nonnegative. Our computations are based on examples given by Meis [2].

2. Meis's work

In [2], Meis demonstrates the existence of special divisors for the case $r = 1$. Since this monograph is rather difficult to obtain, we will review his method in some detail.

His proof proceeds by considering the universal analytic space of special divisors $\mathscr{G}_n^1$ over the Teichmuller space T_g and explicitly exhibiting a special fiber of dimension $\tau + 1$ in the case in which n is the minimum integer such that τ is nonnegative. He may then conclude that a component of $\mathscr{G}_n^1$ has dimension $3g - 3 + \tau + 1$ and that this component maps surjectively down to T_g.

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[1] These results comprise a portion of the author's doctoral dissertation, M.I.T., 1973.