ON THE HIGHER ORDER SECTIONAL CURVATURES

BY

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The riemannian (holomorphic) higher order sectional curvatures are invariants of the riemannian (kaehlerian) structure weaker than the riemannian (holomorphic) sectional curvature. The study of these invariants is very interesting as can be seen by the abundant bibliography on this subject; for example, the articles of Thorpe, Gray, Stehney, Hsiung, Levko, ...

If the riemannian sectional curvature of order two is bounded, Berger [1] gives an estimation of the curvature tensor components. Later, Karcher [2] gives an easy proof of this estimation. We shall prove in Section 1 a generalization of these results to the higher order riemannian curvature tensor components R_p when the sectional curvature of order p is also bounded.

Thorpe [6] gives the characterization of the constancy of the riemannian sectional curvature of order p and he concludes properties on the Pontrjagin classes of these manifolds. In an earlier article [4] we give a characterization of the constancy of the holomorphic sectional curvature of order p and we deduce properties on the Chern classes of the kaehlerian manifolds with constant holomorphic sectional curvature of order two. We shall generalize in Section 2 some results of Thorpe on riemannian sectional curvatures of order p to the holomorphic sectional curvatures of order p and we shall conclude some properties on the Chern classes of the kaehlerian manifolds with constant holomorphic sectional curvature of order p.

1. Higher order curvature tensor estimates

Let M be a riemannian manifold of even dimension n and let $\Lambda^p(M)$ denote the bundle of p-vectors of M. $\Lambda^p(M)$ is a riemannian vector bundle with inner product on the fiber $\Lambda^p(m)$ over m, $m \in M$, related to the inner product on the tangent space M_m of M at m by

$$g(u_1 \wedge \cdots \wedge u_p, v_1 \wedge \cdots \wedge v_p) = \det\{g(u_i, v_j)\}, \quad u_i, v_j \in M_m.$$

Let R denote the covariant curvature tensor of M. For each even $p > 0$ we define the pth curvature tensor R_p of M to be the covariant tensor field of order $2p$ given by

$$\begin{aligned} R_p(u_1, \ldots, u_p, v_1, \ldots, v_p) &= \frac{1}{2^{p/2}p!} \sum_{\alpha, \beta \in S_p} \varepsilon(\alpha)\varepsilon(\beta) R(u_{\alpha(1)}, u_{\alpha(2)}, v_{\beta(1)}, v_{\beta(2)}) \cdots \\ &\qquad R(u_{\alpha(p-1)}, u_{\alpha(p)}, v_{\beta(p-1)}, v_{\beta(p)}) \end{aligned} \tag{1}$$

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