FINITE SIMPLE GROUPS OF 2-RANK 3 WITH ALL 2-LOCAL SUBGROUPS 2-CONSTRAINED

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Introduction

In this paper we obtain the following:

THEOREM. *Let G be a finite simple group of 2-rank 3 in which all 2-local subgroups are 2-constrained. Then G is isomorphic to one of the groups $L_2(8)$, $U_3(8)$, $Sz(8)$, or $G_2(3)$.*

Here to say that G is of 2-rank 3 means that G has an elementary abelian subgroup of order 8 but none of order 16. Alperin, Brauer, and Gorenstein have determined all simple groups of 2-rank 2.

We note also that Stroth has recently obtained this same result using a different method. In addition, Stroth has determined all finite groups of 2-rank 3 in which some 2-local subgroup is not 2-constrained.

The proof of this theorem is possibly more interesting than its statement. One way to prove the theorem is to use a recent theorem of Gorenstein and Lyons [8], to conclude that either G is known, or G possesses a nonsolvable 2-local subgroup H. Set $\bar{H} = H/O(H)$. If 7 divides the order of $\bar{H}$, then a theorem of Alperin yields the structure of $\bar{H}$. Other results then identify G. If 7 does not divide the order of $\bar{H}$, it is possible to show that $\bar{H}/O_2(\bar{H})$ is a subgroup of the automorphism group of A_5 or A_6, and that $O_2(\bar{H})$ is of restricted type. We do not employ this procedure. Rather we prove analogues of Glauberman's ZJ-theorem. Essentially we prove four propositions which guarantee that G has exactly two conjugacy classes of maximal 2-local subgroups. These are:

PROPOSITION 1. *Let H be a 2-constrained group of 2-rank 3 with $O(H) = 1$. Suppose that 7 divides the order of H. Then, either*

(1) *$O_2(H)$ is an abelian group or a Suzuki 2-group and $H/O_2(H)$ is of odd order, or*

(2) *$O_2(H)$ is homocyclic abelian of rank 3, and $H/O_2(H)$ is isomorphic to $L_3(2)$.*

As stated above, known results will identify G if H is a 2-local subgroup of G. When 7 divides the order of no 2-local of G, we wish to obtain a contradiction. The tools of this attempt are the following three propositions.

Received May 20, 1975.

This research was partially supported by a National Science Foundation grant and by the Alfred Sloan Foundation.