# VIRTUAL GROUP HOMOMORPHISMS WITH DENSE RANGE

BY

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## Introduction

Let $\mathscr{F}$ be the virtual subgroup of $Z$ (the integers) defined by an ergodic measure-preserving transformation $\Phi$ on an analytic finite (nonatomic) measure space $X$. A homomorphism $F: \mathscr{F} \to A$ (a locally compact second countable group) has *dense range* if the Anzai skew product transformation on $X \times A$, defined by $F$ and $\Phi$, is ergodic. The main results obtained in this paper are as follows. (a) For every compact second countable group $A$ there exists a homomorphism $F: \mathscr{F} \to A$, with dense range. This responds partially to the problem raised by Mackey (1968) as to whether a virtual subgroup admits a homomorphism "onto" a particular compact group, i.e., with dense range. (b) For every countable abelian group $A$, there exists a homomorphism $F: \mathscr{F} \to A$, with dense range.

The basic approach to obtaining results (a) and (b) is to construct for each group, $A$, a virtual group and strict homomorphism of that virtual group into $A$ with dense range. The virtual groups and homomorphisms all arise from rather conventional examples of free ergodic actions of groups. Then the results of Dye in 1959 and 1963 are applied to show that the virtual groups used for (nontrivial) $A$ compact second countable or countable abelian are all similar to $\mathscr{F}$—in fact, isomorphic (mod i.c.'s). Since the existence of a homomorphism with dense range $F: \mathscr{F} \to A$ (fixed $A$) is a similarity invariant for $\mathscr{F}$ the results (a) and (b) follow immediately.

The results (a) and (b) have interesting consequences in regards to the first cohomology of $\mathscr{F}$, i.e.,

$$H^1(\mathscr{F}; B) = \{\text{homomorphisms } h: \mathscr{F} \to B\} \quad \text{mod similarity,}$$

where the coefficient group $B$ is an analytic Borel group. In [W-1] we prove that if $F: \mathscr{F} \to A$ has dense range then the induced map on the cohomology

$$F^*: H^1(A; B) \to H^1(\mathscr{F}; B)$$

is injective. Hence, using the well-known duality theory for abelian groups (say as in [P]), for $B$ the circle group, $H^1(\mathscr{F}; B)$ contains as subgroups every countable abelian group (by (a)) and every compact abelian second countable group (by (b)).

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